Collapse of the wave field in a system of weakly coupled light guides

A.A. Balakin, A.G. Litvak, V.A. Mironov, S.A. Skobelev

Institute of applied physics RAS, Nizhniy Novgorod, Russia

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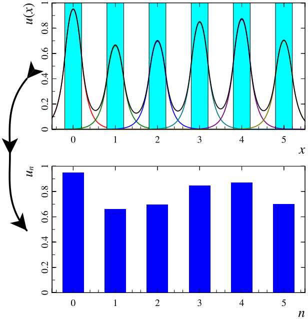
Discrete Shrödinger equation

Discrete NSE

$$\mathrm{i}\frac{\partial u_n}{\partial z} + u_{n+1} - 2u_n + u_{n-1} + |u_n|^2 u_n = 0.$$

It conserve the power

$$\mathcal{P} = \sum_{n=-\infty}^{+\infty} |u_n|^2 = \text{const.}$$



Corresponding Lagrangian is

$$\mathcal{L} = \sum \frac{\mathrm{i}}{2}\left(u_n \frac{\partial u_n^*}{\partial z} - u_n^* \frac{\partial u_n}{\partial z}\right) - u_{n+1}u_n^* - u_{n+1}^* u_n - \frac{1}{2}|u_n|^4.$$

Variational approach (1D case)

Let consider Gaussian wave packets

$$u(x) = \sqrt{\frac{\mathcal{P}}{a\sqrt{\pi}}} \exp\left(-\frac{x^2}{2a^2} + \mathrm{i}\beta x^2\right).$$

Then truncated Lagrangian is

$$\mathcal{L}_c = \frac{d\beta}{dz}\frac{\mathcal{P}a^2}{2} - 2\mathcal{P}\exp\left(-\frac{1}{4a^2} - \beta^2 a^2\right) - \frac{\mathcal{P}^2}{2a}\sqrt{\frac{1}{2\pi}}.$$

That give the equations

$$\frac{d\beta}{dz} = -\frac{\mathcal{P}}{2\sqrt{2\pi}a^3} + \frac{1-4\beta^2 a^4}{2a^2}\frac{2}{a^2}\mathrm{e}^{-\beta^2 a^2 - 1/4a^2},$$

$$\frac{da}{dz} = 4\beta a\ \mathrm{e}^{-\beta^2 a^2 - 1/4a^2}.$$

1D collapse

These equations have the integral

$$\mathcal{C} = \exp\left(-\frac{1}{4a^2} - \beta^2 a^2\right) + \frac{\mathcal{P}}{4\sqrt{2\pi}a}.$$

and can be reduced to:

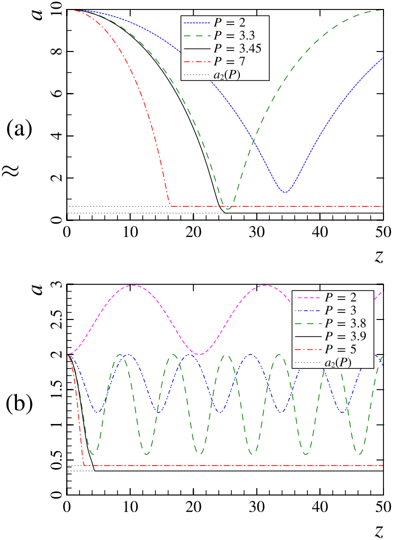
$$\frac{da}{dz} = \pm 4\mathcal{C}\left(1-\frac{a_1}{a}\right)\sqrt{-\ln\left(\mathcal{C}-\frac{\mathcal{C}a_1}{a}\right)-\frac{1}{4a^2}} \approx$$
$$\approx \pm\frac{2\mathcal{C}}{a}\sqrt{\ln\mathcal{C}}\sqrt{(a_0-a)(a-a_2)},$$

where $a_1 = \frac{\mathcal{P}}{4\sqrt{2\pi\mathcal{C}}}$, $a_2 = \frac{\sqrt{2\pi}\mathcal{C}}{\mathcal{P}} - \frac{\mathcal{P}}{8\sqrt{2\pi}\mathcal{C}}$.

This give wave field "collapse" to the only one light-guide for

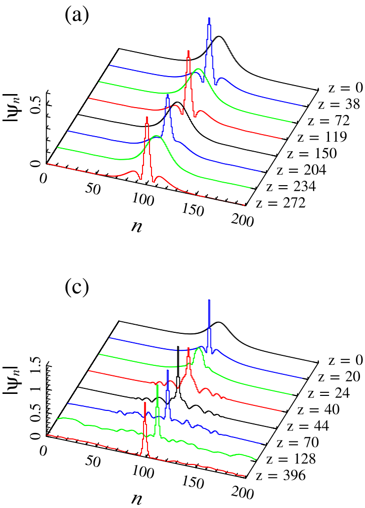
$$\mathcal{P} > \mathcal{P}_{cr} = \sqrt{\frac{48\pi}{11}}\mathcal{C} \approx 3.7\ \mathcal{C}.$$

Numerics give $\mathcal{P}_{cr} \approx 3.3\ \mathcal{C}$.



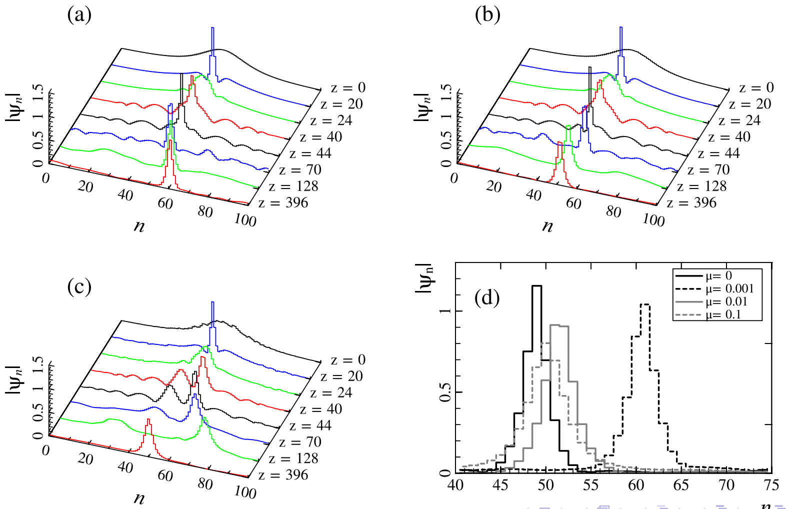
Numerical simulation of 1D collapse

Dynamics of the beam amplitude $|u(z,n)|$ for: (a) $\mathcal{P} = 1.6 < \mathcal{P}_{cr}$, (b) $\mathcal{P} = 1.8 < \mathcal{P}_{cr}$, (c) $\mathcal{P} = 3.6 > \mathcal{P}_{cr}$ and (d) $\mathcal{P} = 10 > \mathcal{P}_{cr}$.



Stability analysis

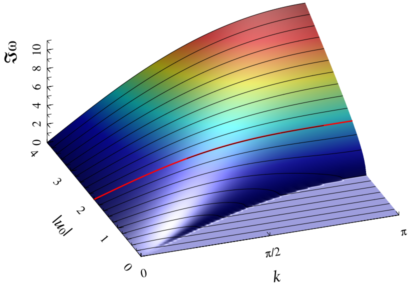
Dynamics of the beam amplitude $|u(z,n)|$ for different noise levels: (a) $\mu = 0.001$, (b) $\mu = 0.01$, and (c) $\mu = 0.1$.



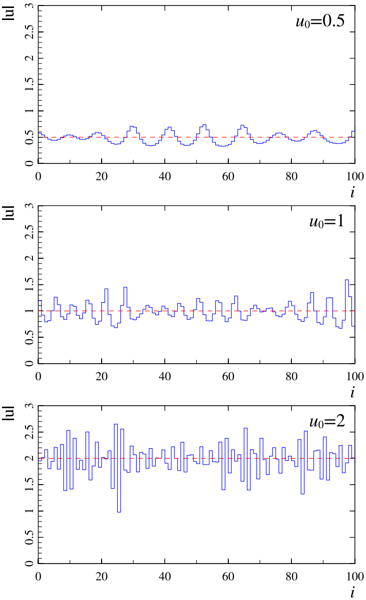
Filamentation instability in discrete media

The increment of filamentation instability for perturbations $\propto \exp(i\kappa n)$ has the form

$$\Im\omega^2 = 4\sin^2\frac{\kappa}{2}\left(2u_0^2 - 4\sin^2\frac{\kappa}{2}\right).$$



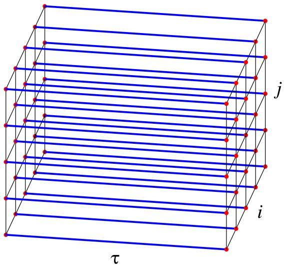
It is maximal at the lattice period $L_\perp \equiv \pi/\kappa = 1$ for $|u_0| \geq 2$.



2D set of light-guides

Lets extend dimensions

$$\mathrm{i}\frac{\partial u_{m,n}}{\partial z} + u_{m+1,n} - 2u_{m,n} + u_{m-1,n} + \\ + u_{m,n+1} - 2u_{m,n} + u_{m.n-1} + \\ + \gamma\frac{\partial^2 u_{m,n}}{\partial \tau^2} + |u_{m,n}|^2 u_{m,n} = 0.$$



It conserve the power

$$\mathcal{P} = \sum \int |u_{mn}|^2 d\tau = \text{const.}$$

Corresponding Lagrangian is

$$\mathcal{L} = \sum \frac{\mathrm{i}}{2}\left(u_{mn}\frac{\partial u^*_{mn}}{\partial z} - u^*_{mn}\frac{\partial u_{mn}}{\partial z}\right) + \gamma\left|\frac{\partial u_{mn}}{\partial \tau}\right|^2 - \frac{1}{2}|u_{mn}|^4 - \\ - u_{m,n+1}u^*_{mn} - u^*_{m,n+1}u_{mn} - u_{m+1,n}u^*_{mn} - u^*_{m+1,n}u_{mn}.$$

Variational approach (3D case)

Let consider Gaussian wave packets

$$u_{n,m} = \frac{\sqrt{W}}{a\sqrt{\tau_0}\sqrt[4]{\pi^3}} \exp\left(-\frac{\tau^2}{2\tau_0^2} - \frac{(x^2+y^2)}{2a^2} + i\beta(x^2+y^2) + i\alpha\tau^2\right).$$

Then truncated Lagrangian is ($\sigma = 4\pi\sqrt{2\pi}$)

$$\mathcal{L}_c = \dot{\beta}a^2W + \frac{1}{2}\dot{\alpha}\tau_0^2 W + \frac{W}{2\tau_0^2}\left(1+4\alpha^2\tau_0^4\right) - \\ - 4W\exp\left(-\frac{1}{4a^2} - \beta^2a^2\right) - \frac{W^2}{\sigma a^2 \tau_0}.$$

That give the equations

$$\dot{\beta} = \left(\frac{1}{a^4} - 4\beta^2\right) e^{-\frac{1}{4a^2} - \beta^2 a^2} - \frac{W}{\sigma a^4 \tau_0},$$

$$\dot{a} = 4\beta a e^{-\frac{1}{4a^2} - \beta^2 a^2},$$

$$\ddot{\tau}_0 = \frac{4}{\tau_0^3} - \frac{4}{\sigma}\frac{W}{a^2\tau_0^2}.$$

Discrete collapse in 2D case

For long pulses $\tau_0 \gg a$, these equations have integral ($\mathcal{P} \equiv \frac{W}{\sqrt{2\pi}\tau_0}$)

$$\mathcal{C} = \exp\left(-\frac{1}{4a^2} - \beta^2 a^2\right) + \frac{\mathcal{P}}{16\pi a^2} \quad \Rightarrow \quad a_{\min} = \sqrt{\frac{\mathcal{P}}{16\pi\mathcal{C}}},$$

and can be reduced to:

$$\frac{da}{dz} = \pm 4\left(\mathcal{C} - \frac{\mathcal{P}}{16\pi a^2}\right)\sqrt{-\ln\left(\mathcal{C} - \frac{\mathcal{P}}{16\pi a^2}\right) - \frac{1}{4a^2}}.$$

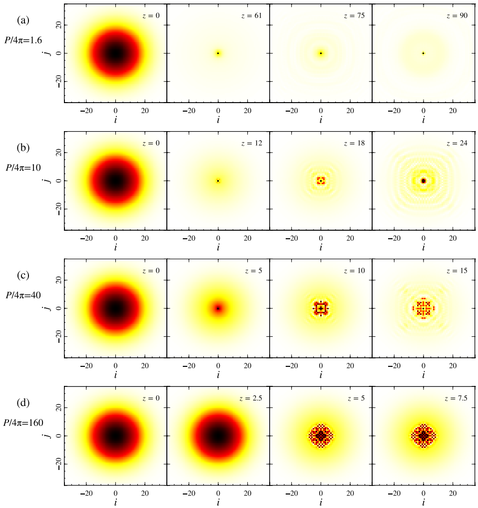
For $a \gg a_{\min}$, the wave field collapse

$$\frac{da}{dz} \approx -\frac{1}{\sqrt{\pi}a}\sqrt{\mathcal{P} - 4\pi} \quad \Rightarrow \quad z^* = \frac{a_0^2\sqrt{\pi}}{2\sqrt{\mathcal{P} - 4\pi}}.$$

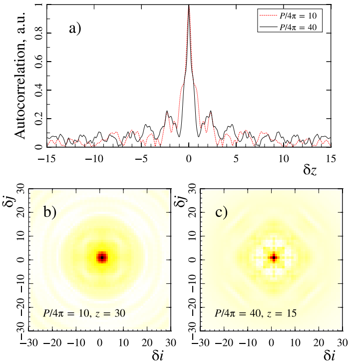
At the final stage, the wave beam width decrease to the minimum size $a_{\min}$ by the asymptotic law

$$a \approx a_{\min}\left[1 + \frac{1}{2}\exp\left(-\frac{z^2}{a_{\min}^2}\right)\right].$$

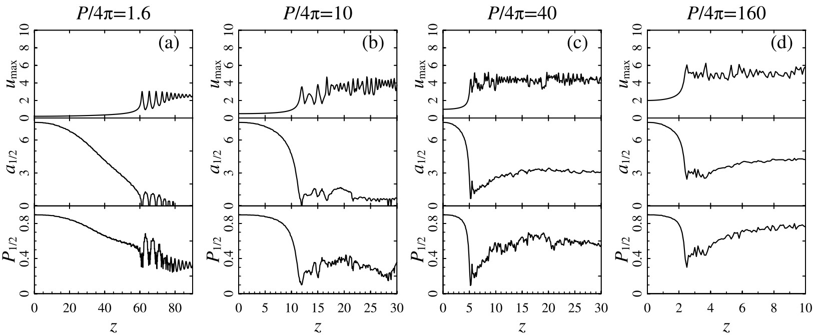
Numerical simulation (2D case)



The amplitude $|u| \lesssim \sqrt{2\pi}$ is limited by phase synchronism $L_d\delta\omega \simeq \frac{1}{2}|u|^2 \ll \pi$. But nonlinearity "focus" wave beam at edges. So, dynamics become stochastic for $\mathcal{P}/4\pi \ggg 1$.



Collapse characteristics



- The collapse is possible for powers about the critical one.
- The field amplitude is the limited and very non-regular for higher $\mathcal{P}$.
- The width of strong field area $a_{1/2}$ is larger for higher powers.
- The noticeable part of power is contained in strong field area.

Laser pulse self-compression

For short pulses $\tau_0 \ll a$, the "slow" motion law is

$$\tau_0(z) \approx \frac{\sigma}{W} a^2(z) \equiv \frac{a^2(z)}{a_c^2}, \quad a_c = \sqrt{W/\sigma}.$$

As result, the equations have the integral

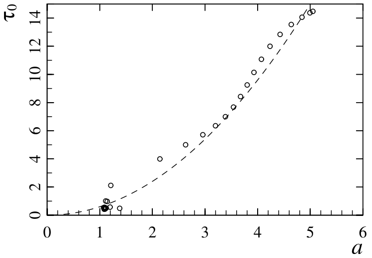
$$\mathcal{C} = \exp\left(-\frac{1}{4a^2} - \alpha^2 a^2\right) + \frac{W^2}{2\sigma^2 a^4}.$$

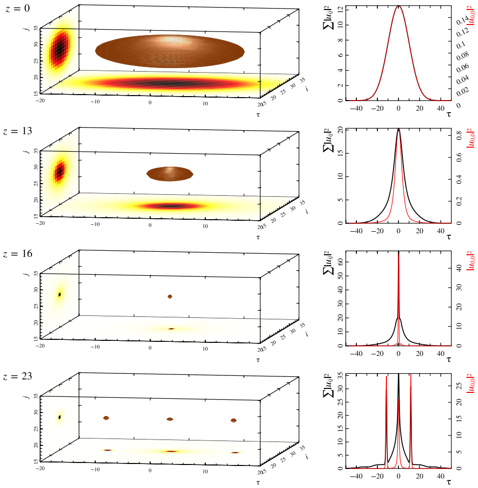
For wide beams $a \gg a_c$, it looks as in continuous media: the collapse occur for $W > W_{\text{cr}} = \sqrt{2}\sigma a_0$. At the final stage, the wave packet width and duration decrease by the asymptotic law

$$\tau_0 = \left(\frac{a}{a_c}\right)^2 = \left[1 + \frac{1}{4} e^{-64z^2/a_c^2}\right]^2.$$

So, radiation self-focusing is accompanied by the noticeable shortening of the duration of 3D wave packets with a soliton-like distribution along the longitudinal coordinate.

Numerical simulation of Gaussian pulse



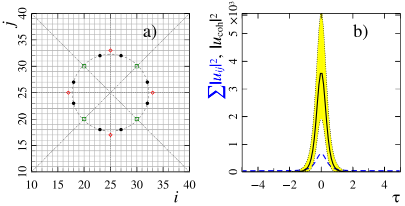


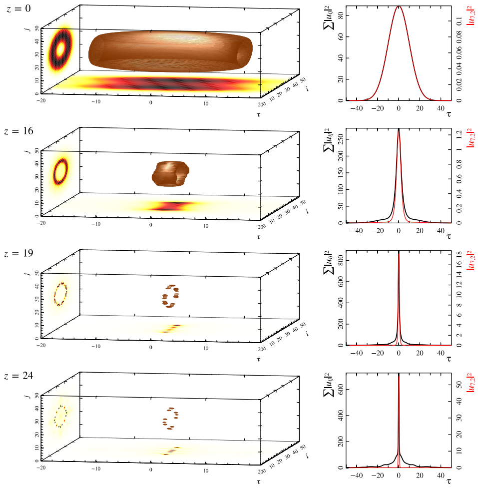
Indeed, the numerical simulation confirm the qualitative study at the beginning.

However, modulation instability at the final stage stops the shortening and breaks the pulse on the several ones.

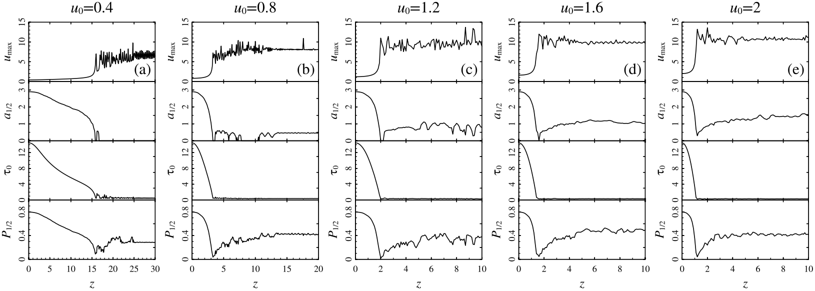
Numerical simulation of hollow pulse

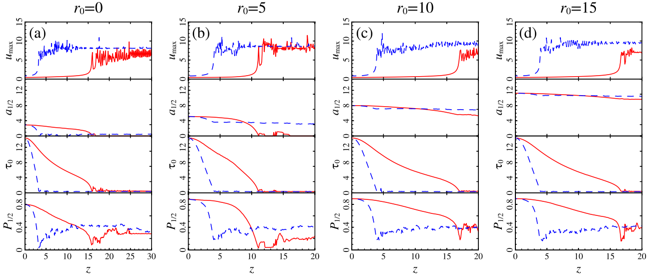
We suggest to use hollow wave packets for avoiding modulation. There is region on the plane of field amplitudes and ring radius, where transverse filamentation is dominant. Thereat, a set of light bullets is formed in the same transverse plane. Moreover some of them a coherent.





Integral characteristics





Conclusion

- The "collapse" in one-dimensional discrete media is shown.
- There is limitation for field amplitude both in 1D and 2D cases due to the filamentation instability in discrete media.
- The analogue of continuous self-compression mode is found for discrete media too.
- It is shown that self-compression of hollow laser pulses will produce the coherent set of light bullets.