SOLITONS, COLLAPSES AND TURBULENCE
Chernogolovka, 20-25 May 2017

Nonlinear localized flat-band modes in pseudo-spinor diamond chain

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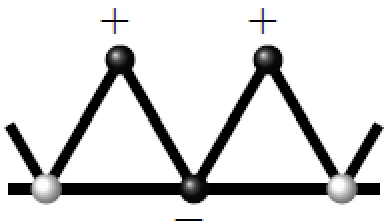
Outline

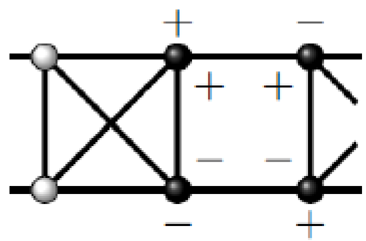
- Flat-band (FB) systems
- Diamond chain
- Diamond chain with spin-orbit coupling (SOC)
- Nonlinear diamond chain with SOC
- Summary

FB systems

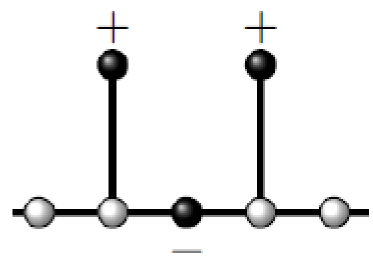
Specific Local Symmetries → **Dispersionless FBs with Compact Localized States (CLSs)**



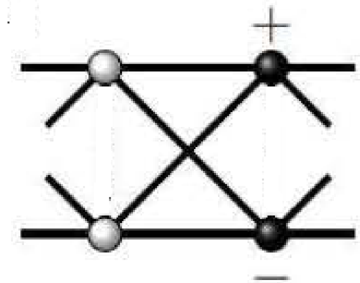
sawtooth



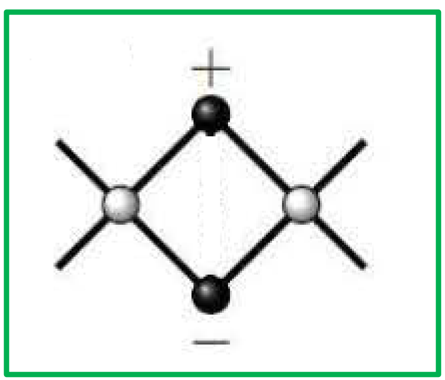
pyrochlore



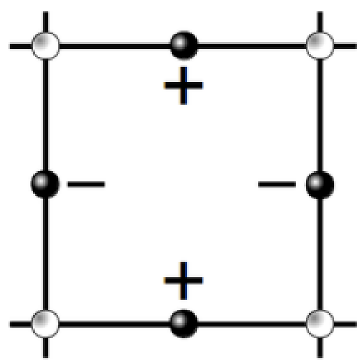
stub



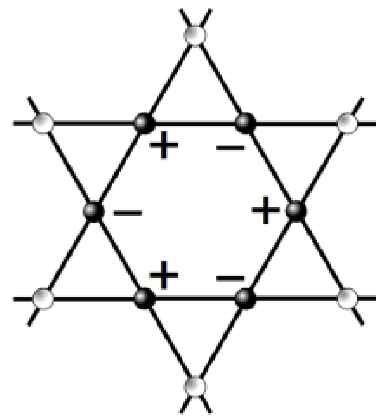
cross-stich



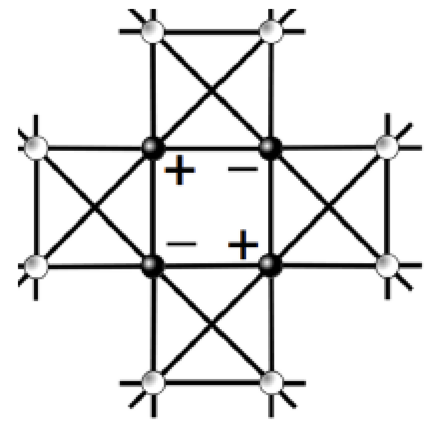
diamond



Lieb



kagome



pyrochlore 2D

BEC playground for nl dynamics

Mean field approximation
Gross-Pitaevski equation

$$\hat{H}_{COL} = \gamma \left| \Psi(\mathbf{r},t) \right|^2$$

Trapping potential

Spin-orbit coupling

Collisions

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{TR}(\vec{r}) + V_{OL}(\vec{r}) + \hat{H}_{SOC} + \hat{H}_{DD} + \hat{H}_{COL} + \ldots \right] \Psi$$

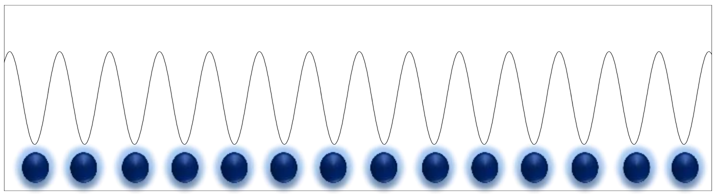
Optical lattice

Dipol-dipol interaction

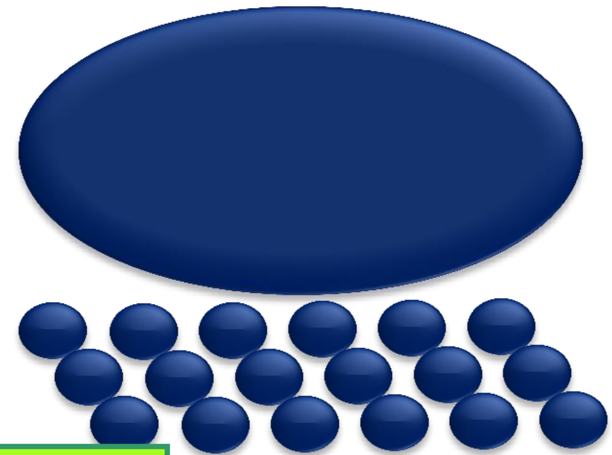
$$\hat{H}_{DD} = \int \left| \Psi(\mathbf{r}',t) \right|^2 V_{dd}(\mathbf{r}-\mathbf{r}') d\mathbf{r}'$$

BEC playground for nl dynamics

1D cigar shaped



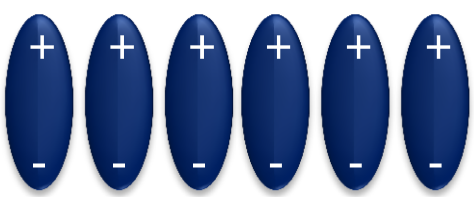
2D cigar pancake

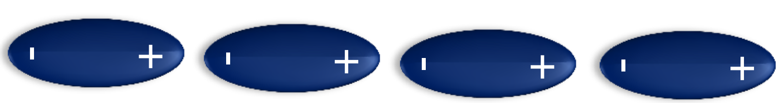


$$V_{dd} = \gamma_{dd} \frac{(\mathbf{e}_1 \cdot \mathbf{e}_2) r^2 - 3(\mathbf{e}_1 \cdot \mathbf{r})(\mathbf{e}_2 \cdot \mathbf{r})}{r^5}$$

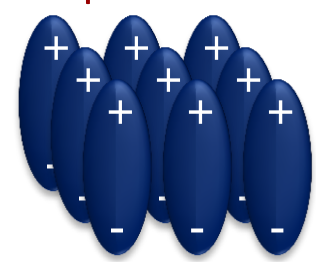
$$V_{dd} = \gamma_{dd} \frac{1 - 3\cos^2\theta}{r^3}$$

DD interaction

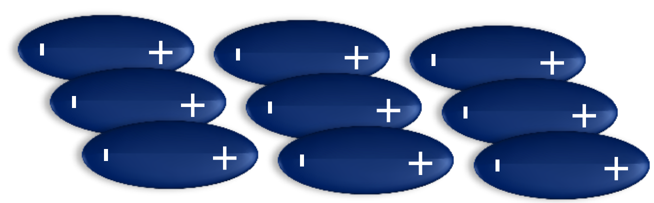




Isotropic DD interaction



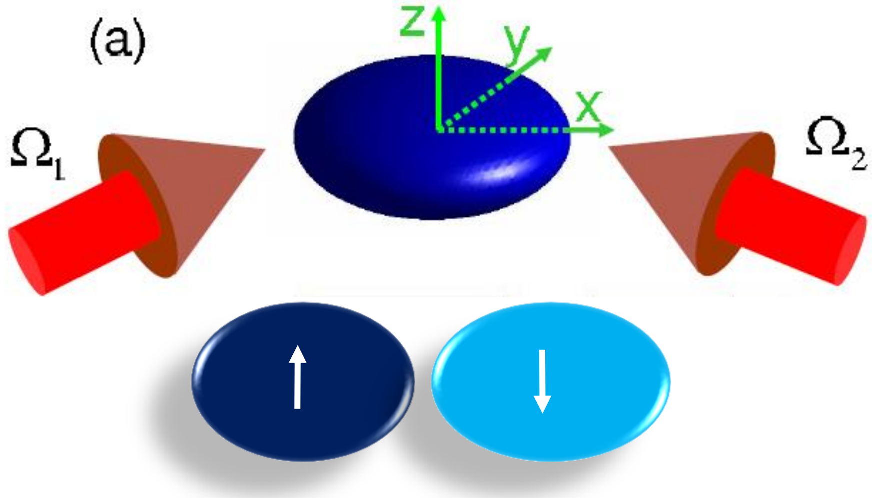
Anisotropic DD interaction

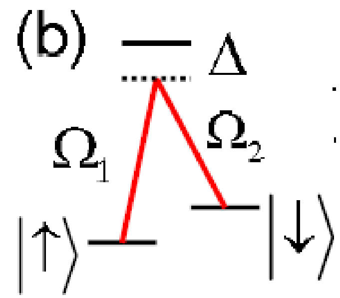


Pseudo-spinor BEC with SOC

BEC pseudo-spinor wave function: ^{87}Rb $5S_{1/2}$, F=1

Synthetic SOC emulating two spin states: $|\Psi_{+}\rangle = |F=1, m_F=0\rangle$, $|\Psi_{-}\rangle = |F=1, m_F=-1\rangle$





Zeeman spliting:

$$\hat{H}_{ZS} = \Delta\hat{\sigma}_z$$

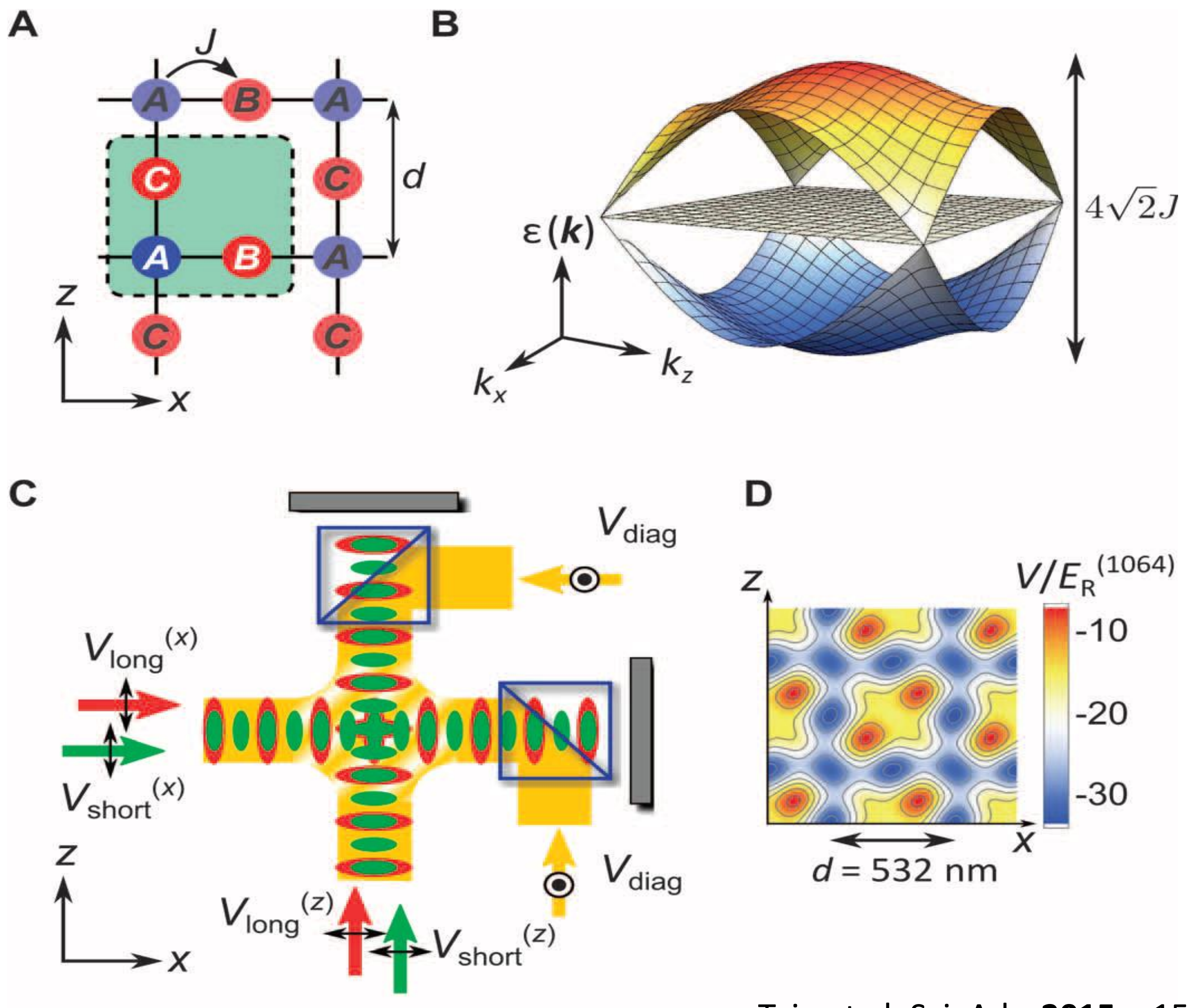
Rashba SOC: $\hat{H}_{SOC} = \lambda\left(i\hbar\frac{\partial}{\partial y}\hat{\sigma}_x - i\hbar\frac{\partial}{\partial x}\hat{\sigma}_y\right)$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$i\hbar\frac{\partial}{\partial t}\begin{bmatrix} \Psi^{+} \\ \Psi^{-} \end{bmatrix} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{OL}(\vec{r}) + \hat{H}_{SOC} + \hat{H}_{ZS} + \hat{H}_{\text{int}}\right]\begin{bmatrix} \Psi^{+} \\ \Psi^{-} \end{bmatrix}$$

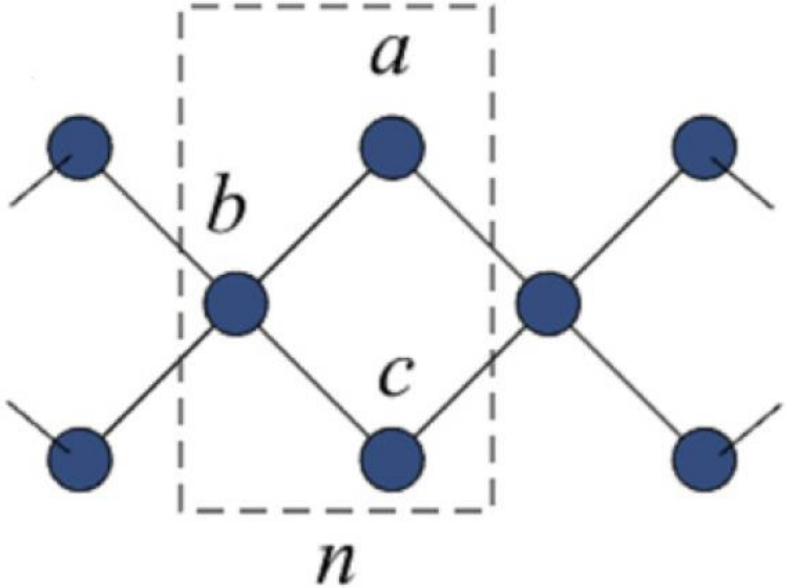
Lin et al. *Nature* **471**, 83 (2011)

FB systems in BEC experimental realization



Taie et al. Sci. Adv. **2015**, e1500854 (2015)

Diamond chain



$$i\frac{da_n}{dt} + \alpha(b_n + b_{n+1}) + \boxed{\beta|a_n|^2 a_n} = 0$$

$$i\frac{db_n}{dt} + \alpha(a_n + a_{n-1} + c_n + c_{n-1}) + \boxed{\beta|b_n|^2 b_n} = 0$$

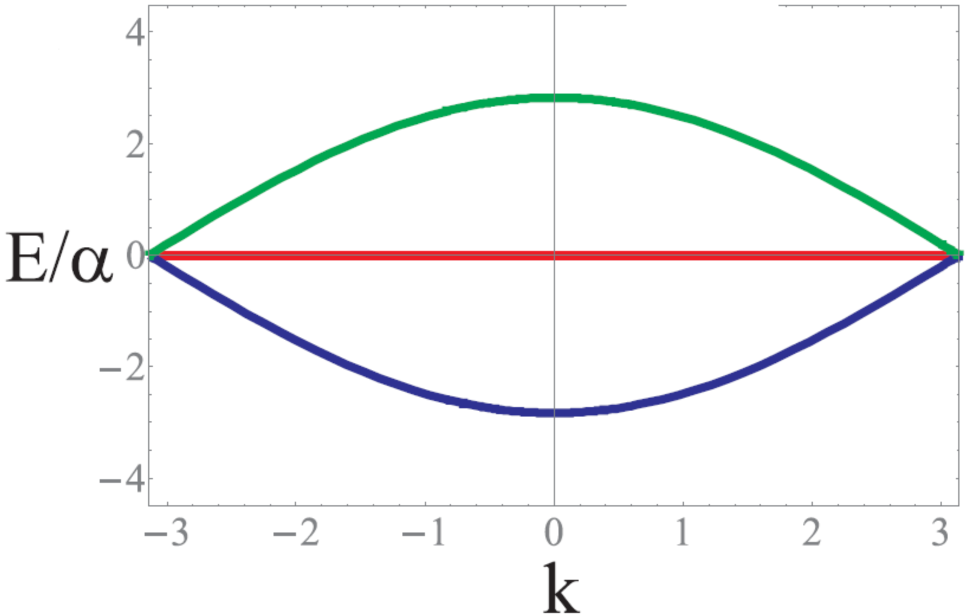
$$i\frac{dc_n}{dt} + \alpha(b_n + b_{n+1}) + \boxed{\beta|c_n|^2 c_n} = 0$$

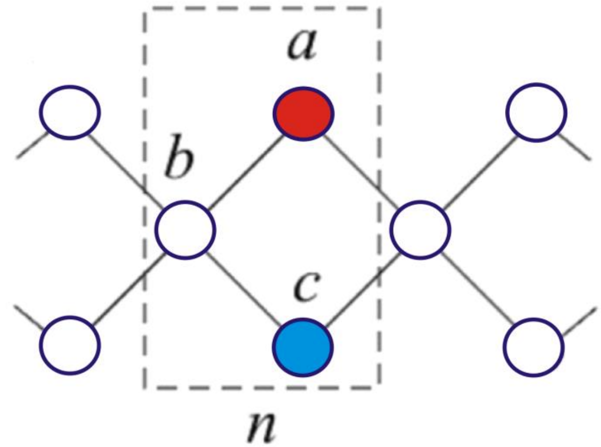
nonlinear interactions

Diamond chain

Linear case: $\beta = 0$

Bloch basis: $\Psi_n = \Psi e^{ikn}$





$$\{a_n(t), b_n(t), c_n(t)\} = \Psi_n e^{-iEt}$$

Dispersive Bands

$$E_{\pm}(k) = \pm 2\sqrt{2}\alpha \cos(k/2)$$

Extended eigenvecotrs:

$$\Psi^{(\pm)} = \frac{1}{2}\left(1, \pm \frac{1 + e^{ik}}{\sqrt{1 + \cos k}}, 1\right)$$

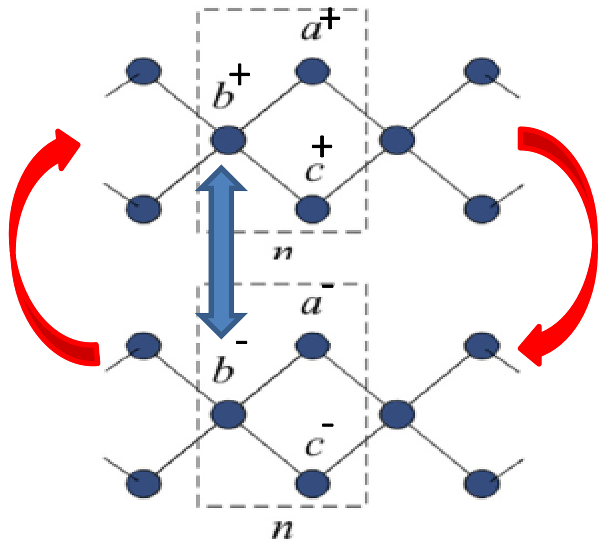
Flat-Bands

$$E_{\mathrm{FB}}(k) = 0$$

Compact Localized States (CLSs):

$$\Psi_n = \delta_{n,n_0}\Psi^{(0)} \equiv \delta_{n,n_0}\frac{1}{\sqrt{2}}(1, 0, -1)$$

Diamond chain with SOC



$$i\frac{da_n^+}{dt} + Ba_n^+ + b_n^+ + b_{n+1}^+ + \lambda(b_{n+1}^- + ib_n^-) + (\gamma|a_n^+|^2 + \zeta|a_n^-|^2)a_n^+ = 0,$$

$$i\frac{db_n^+}{dt} + Bb_n^+ + a_n^+ + a_{n-1}^+ + c_n^+ + c_{n-1}^+ + \lambda[c_n^- - a_{n-1}^- - i(a_n^- - c_{n-1}^-)] + (\gamma|b_n^+|^2 + \zeta|b_n^-|^2)b_n^+ = 0$$

$$i\frac{dc_n^+}{dt} + Bc_n^+ + b_n^+ + b_{n+1}^+ - \lambda(b_n^- + ib_{n+1}^-) + (\gamma|c_n^+|^2 + \zeta|c_n^-|^2)c_n^+ = 0$$

$$i\frac{da_n^-}{dt} - Ba_n^- + b_n^- + b_{n+1}^- - \lambda(b_{n+1}^+ - ib_n^+) + (\gamma_1|a_n^-|^2 + \zeta|a_n^+|^2)a_n^- = 0$$

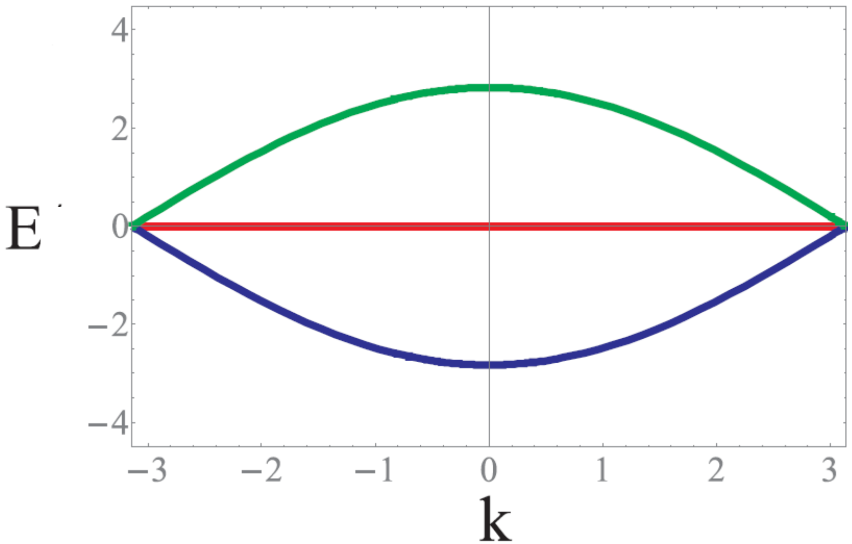
$$i\frac{db_n^-}{dt} - Bb_n^- + a_n^- + a_{n-1}^- + c_n^- + c_{n-1}^- - \lambda[c_n^+ - a_{n-1}^+ + i(a_n^+ - c_{n-1}^+)] + (\gamma_1|b_n^-|^2 + \zeta|b_n^+|^2)b_n^- = 0$$

$$i\frac{dc_n^-}{dt} - Bc_n^- + b_n^- + b_{n+1}^- + \lambda(b_n^+ - ib_{n+1}^+) + (\gamma_1|c_n^-|^2 + \zeta|c_n^+|^2)c_n^- = 0$$

Zeeman term　　Rashba SOC　　nonlinear interactions

Linear case: $\gamma=0$

Single component BEC (λ=0,ζ=0)



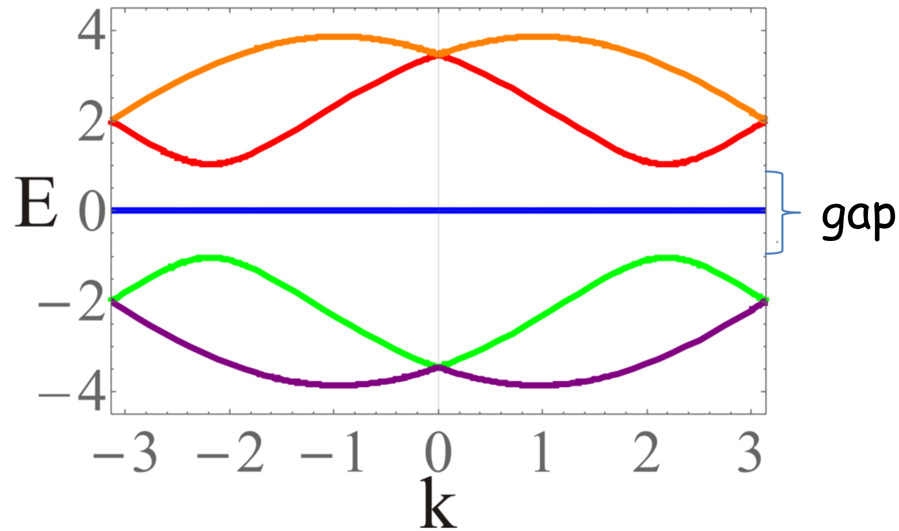
Flat-Bands

$$E_{\mathrm{FB}}(k) = 0$$

Dispersive Bands

$$E_{\pm}(k) = \pm 2\sqrt{2}\alpha \cos(k/2)$$

Pseudo-spinor BEC:



Flat-Bands

$$E_{1,2} = 0$$

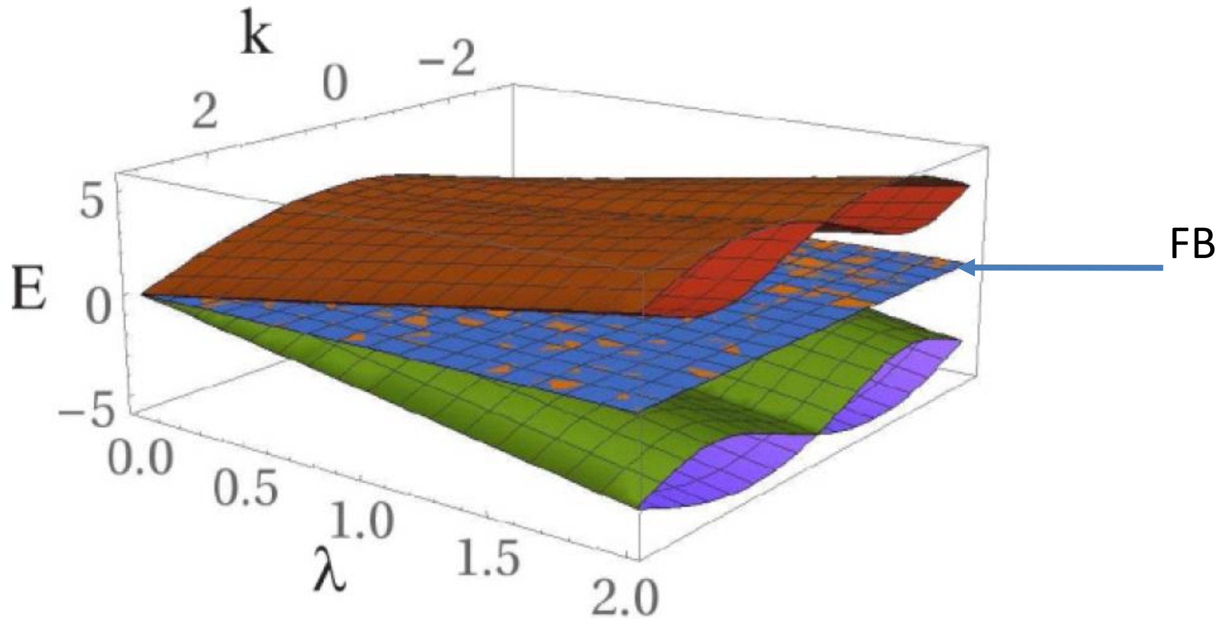
Dispersive Bands

$$E_{3,4} = \pm 2\sqrt{1 + \lambda^2 + \cos k - \sqrt{2}\lambda |\sin k|}$$

$$E_{5,6} = \pm 2\sqrt{1 + \lambda^2 + \cos k + \sqrt{2}\lambda |\sin k|}$$

Pseudo-spinor BEC

SOC opens (mini) gap between FB and DB



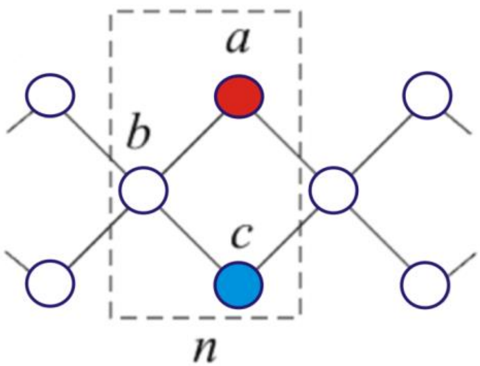
$$W_{\rm mg}(\lambda) = 2\sqrt{1+\lambda^2-\sqrt{1+2\lambda^2}}$$

Single component BEC ⟶ Pseudo-spinor BEC: $\gamma,\zeta=0$

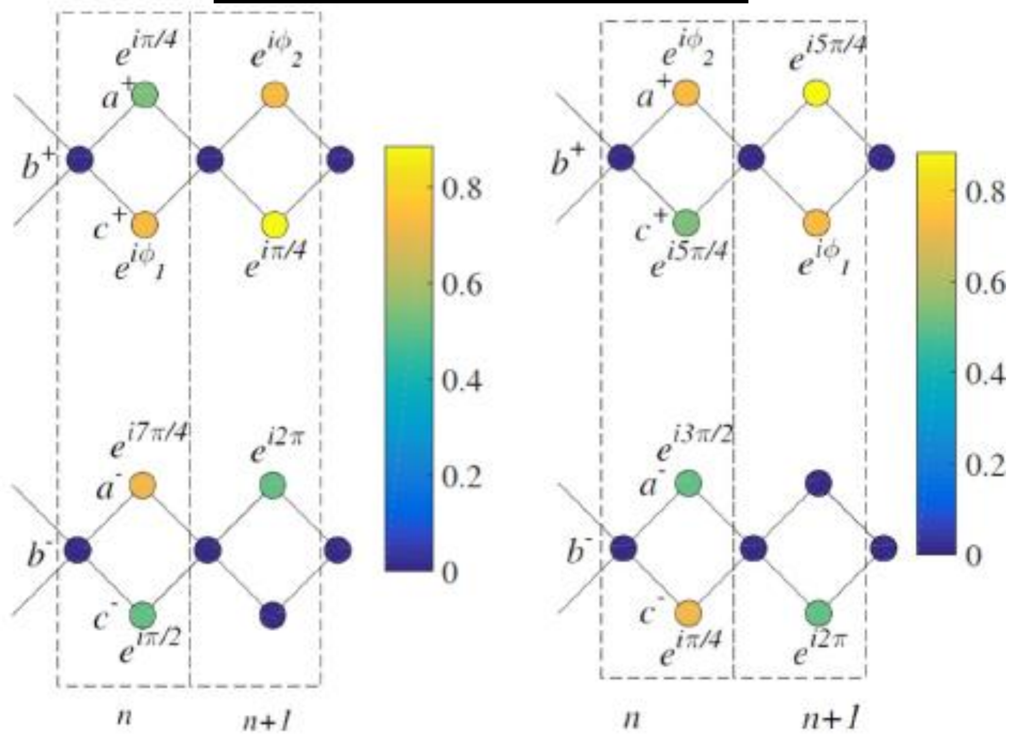
$\gamma=0$

FB CLSs, class U=2:

FB CLSs, class U=1:



$$\Psi_n = \delta_{n,n_0}\Psi^{(0)} \equiv \delta_{n,n_0}\frac{1}{\sqrt{2}}(1,0,-1)$$



Total norm

$$N = \sum_n |\Psi_n|^2 = 2|C|^2(\lambda^2+1)^2$$

Participation number

$$P_t = \frac{4(\lambda^2+1)^4}{(1/4)[(\lambda^2-1)^4+(\lambda^2+1)^4]+(1/2)(\lambda^4+1)^2+6\lambda^4}$$

$$\Psi_n^{(1)} = C\{\delta_{n,n_0}\left(\frac{\lambda^2-1}{2}(1+i), 0, \frac{\lambda^2+1}{2} - i\frac{\lambda^2-1}{2}, \lambda(1-i), 0, i\lambda\right) + \delta_{n+1,n_0}\left(\frac{\lambda^2-1}{2} - i\frac{\lambda^2+1}{2}, 0, \frac{\lambda^2+1}{2}(1+i), \lambda, 0, 0\right)\},$$

$$\Psi_n^{(2)} = C\{\delta_{n,n_0}\left(\frac{\lambda^2-1}{2} - i\frac{\lambda^2+1}{2}, 0, -\frac{\lambda^2-1}{2}(1+i), -i\lambda, 0, \lambda(1+i)\right) + \delta_{n+1,n_0}\left(-\frac{\lambda^2+1}{2}(1+i), 0, \frac{\lambda^2+1}{2} - i\frac{\lambda^2-1}{2}, 0, 0, \lambda\right)\},$$

Nonlinear in diamond chain with SOC

The **CLSs** survive only if both the self– and cross-interactions are present and the corresponding nonlinearity parameters are equal:

$$\gamma = \gamma_1 = \zeta$$

$$|E| = \frac{\gamma}{2}|C|^2\left(\lambda^2 + 1\right)^2 \qquad N = 4\frac{|E|}{\gamma}$$

- **CLSs** coexist with the discrete solitons (**DSs**) in a mini-gap
- **DSs** in a semiinfinite gap (SIG)

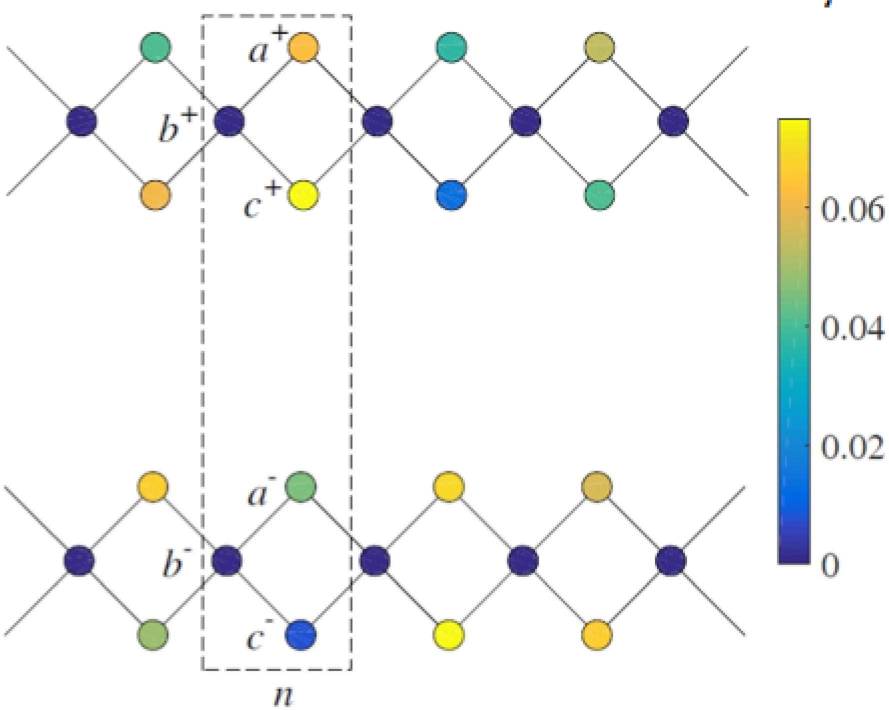
Nonlinear diamond chain with SOC

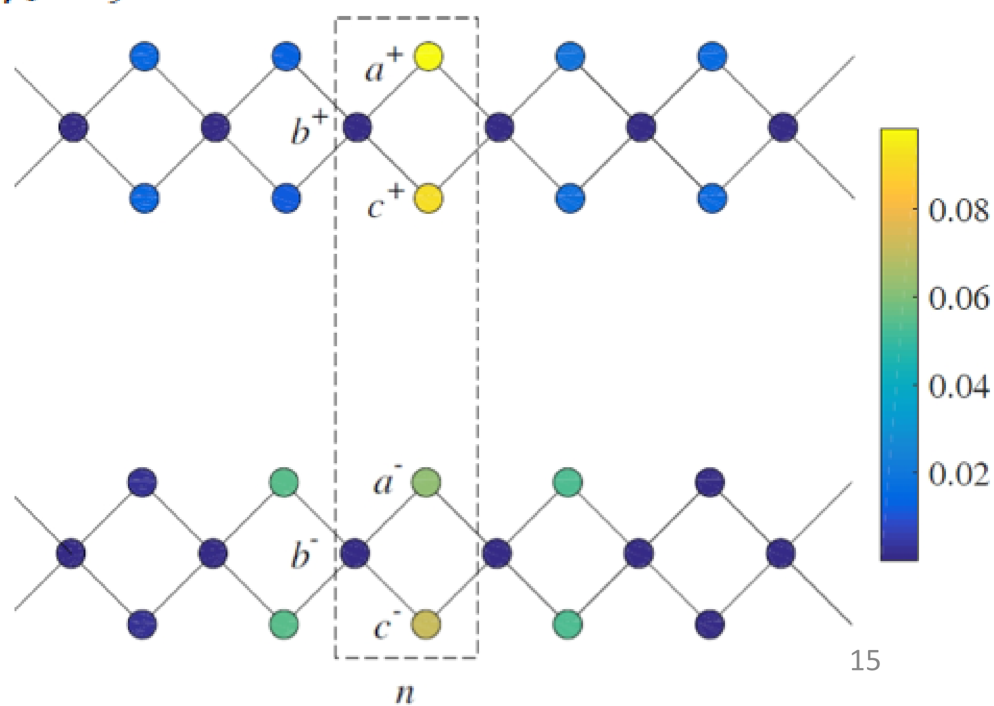
The CLSs survive only if both the self– and cross-interactions are present and the corresponding nonlinearity parameters are equal: $\gamma = \gamma_1 = \zeta$

$$|E| = \frac{\gamma}{2}|C|^2\left(\lambda^2 + 1\right)^2 \qquad N = 4\frac{|E|}{\gamma}$$

Discrete Solitons (DSs) in mini-gap

$$\gamma = \gamma_1 = \zeta$$

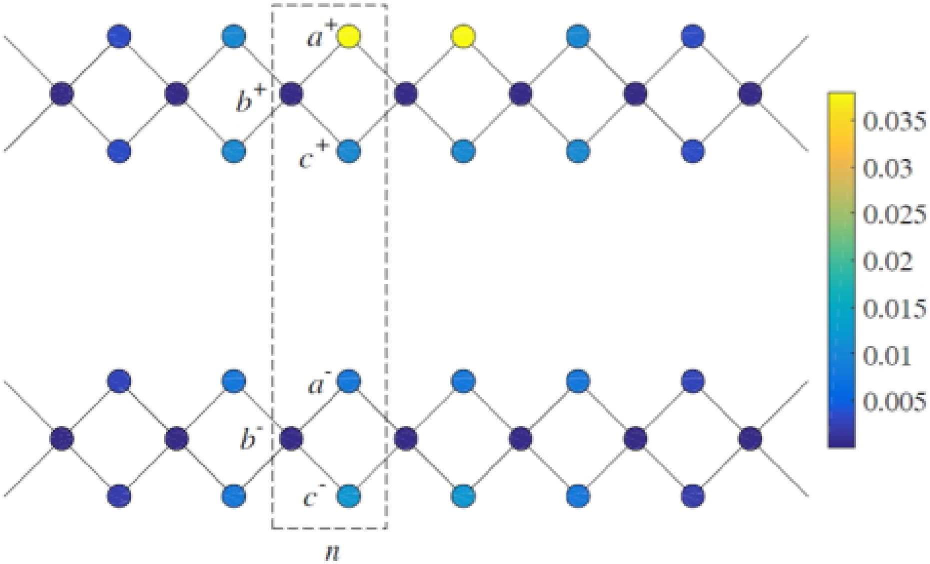


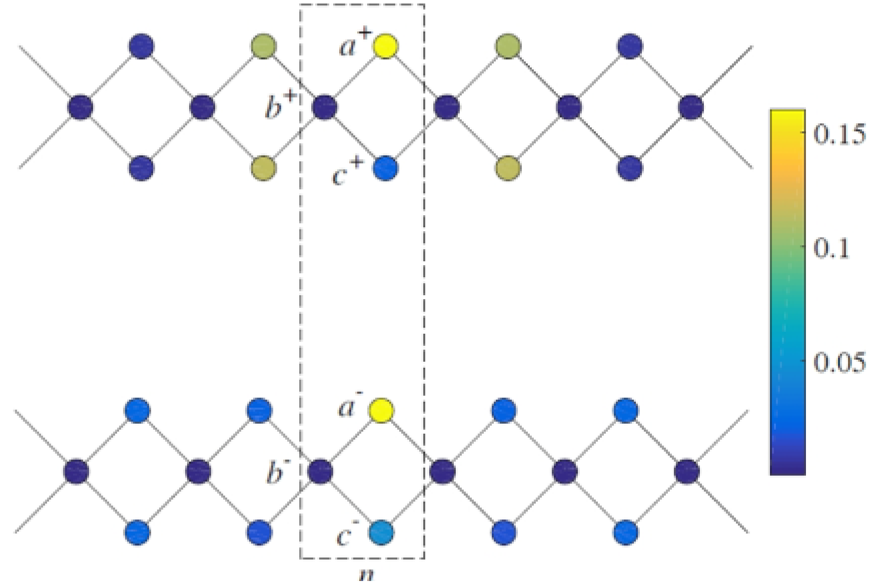


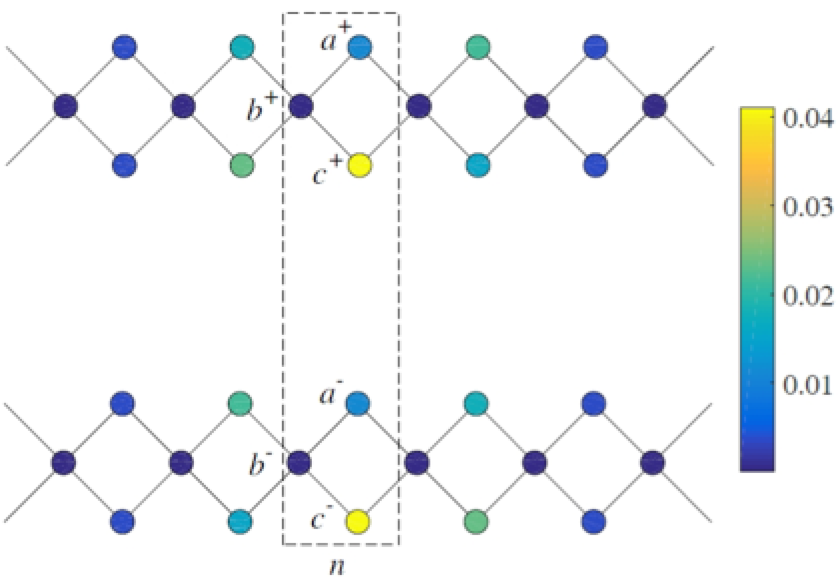
Nonlinear diamond chain with SOC

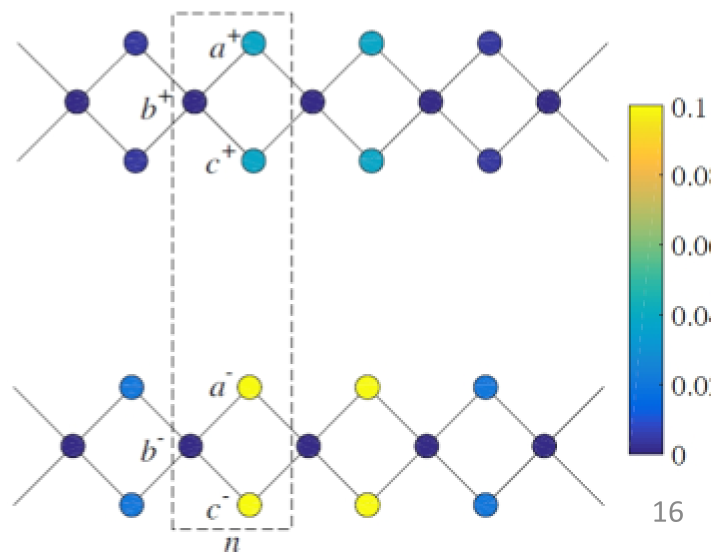
Discrete Solitons (DSs) in mini-gap

$$\gamma = \gamma_1 \quad \zeta = 0$$

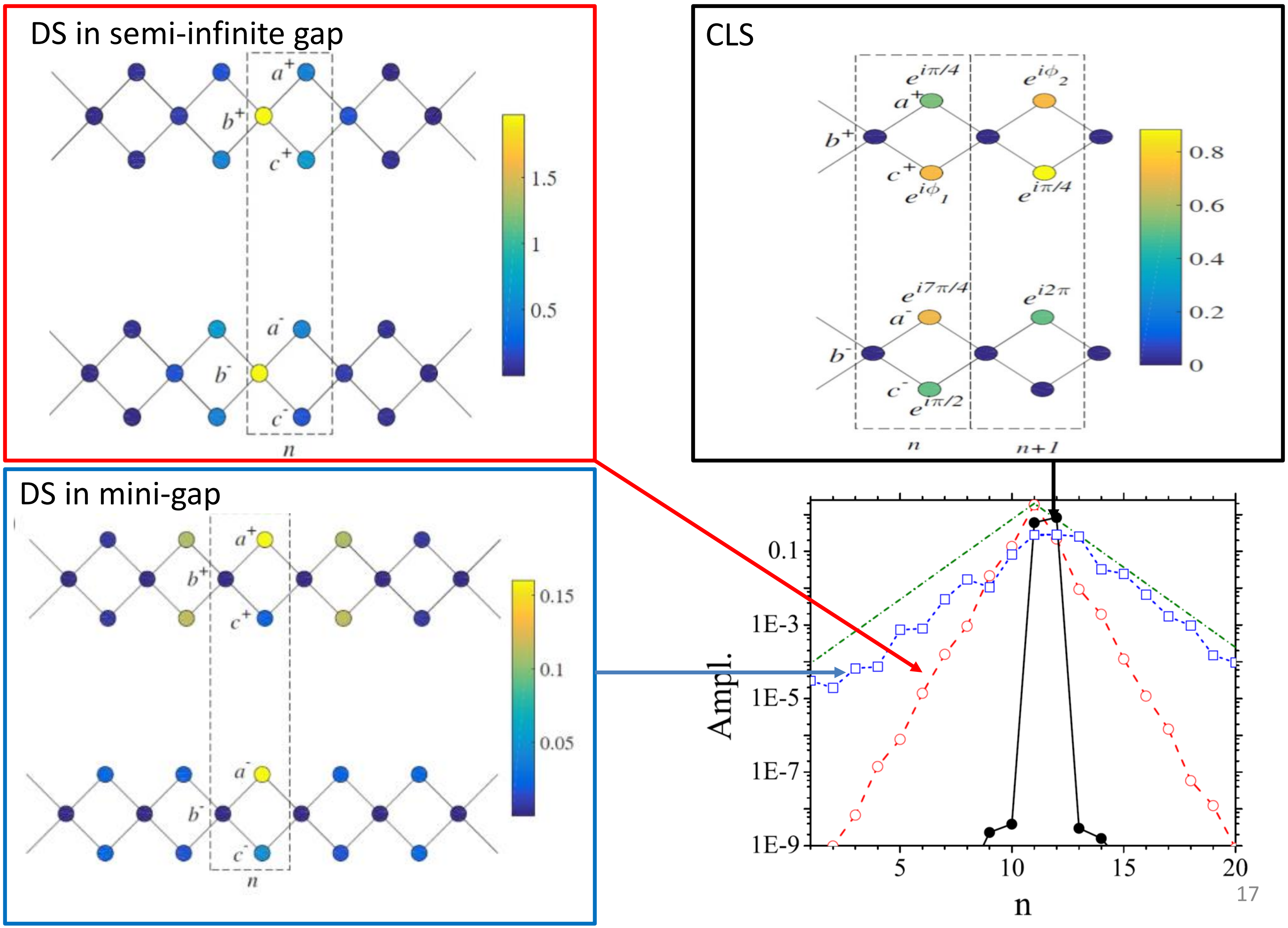








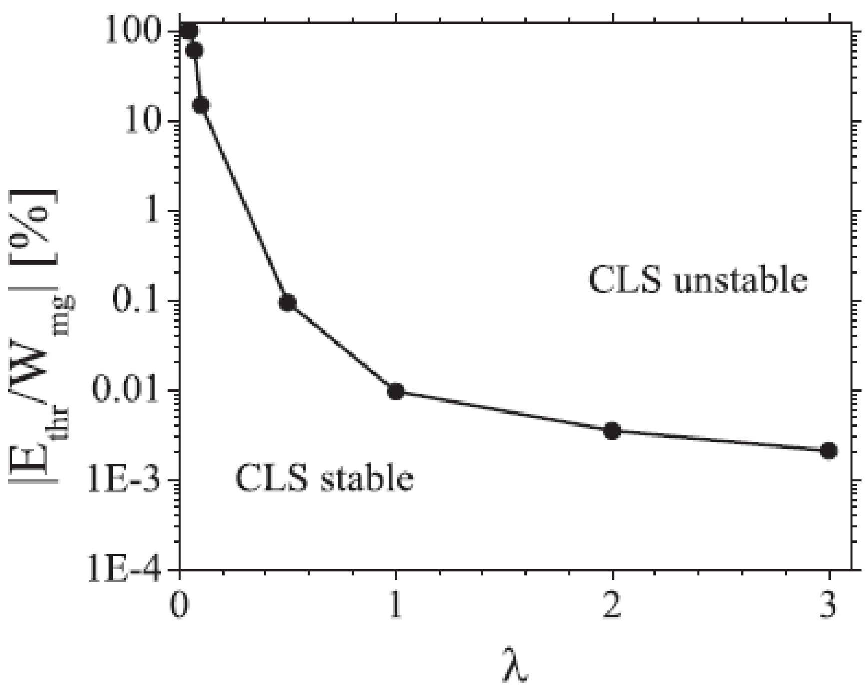
Nonlinear diamond chain with SOC



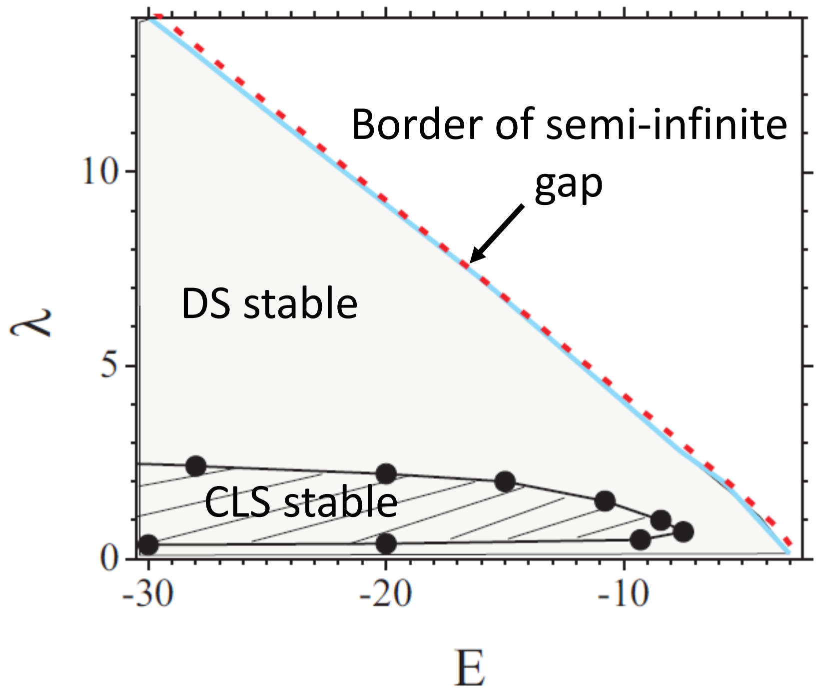
Nonlinear diamond chain with SOC

Stability of localized modes

Mini-gap

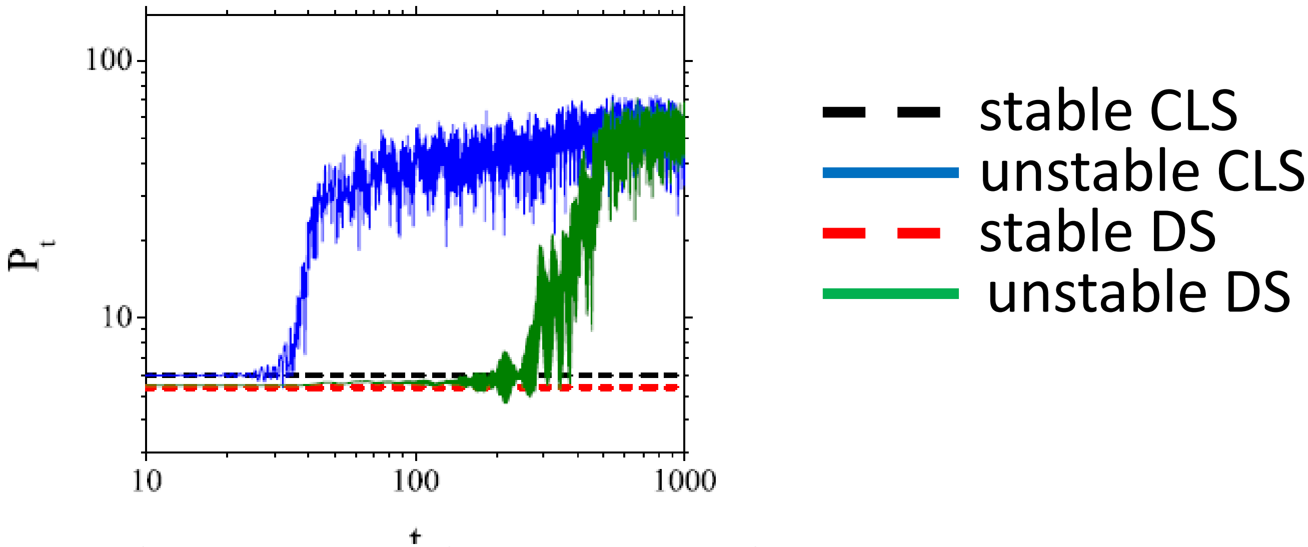


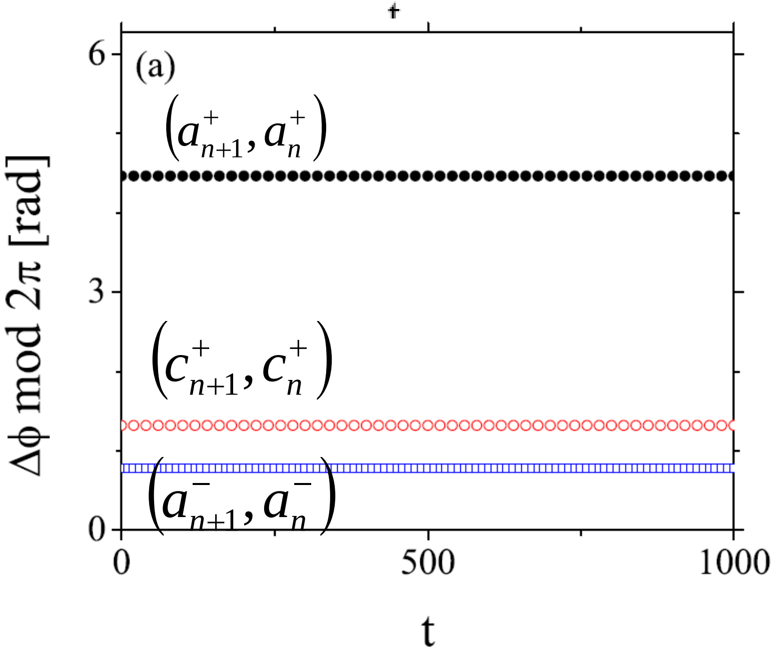
Semi-infinite gap

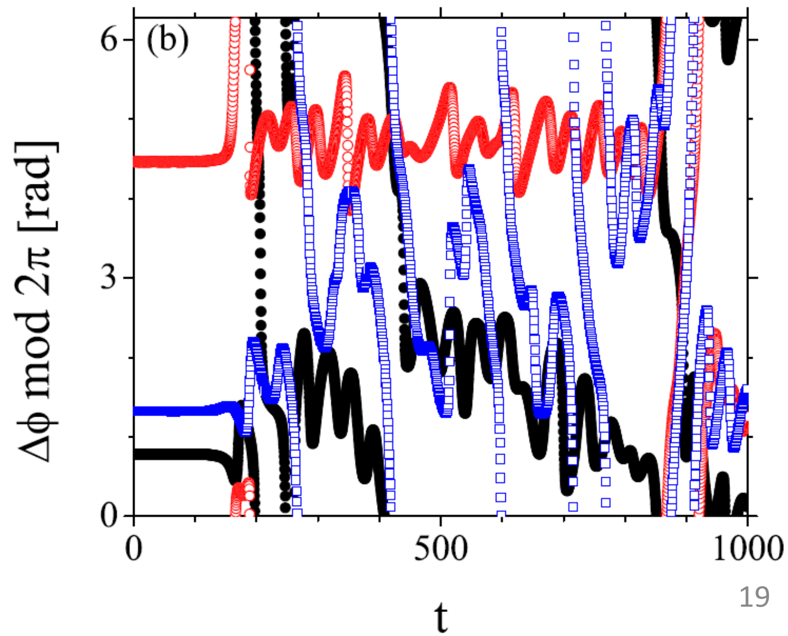


Nonlinear diamond chain with SOC

Stability of localized modes







Summary

- FB CLSs of class U=1 are eigenmodes in linear diamond chain without SOC
- SOC opens a gap between FB and DBs ➔ CLSs is of class U=2
- Nonlinear CLSs persist with frequencies smoothly tuned into the gap
- DSs and CLSs coexist in gaps; stability the vicinity of the (linear) FB
- Inside SIG the CLSs and DSs coexist and can be stable
- Initial conditions determine which localized mode will be realized – CLS of DS.

Physical Review B 94, 144302 (2016)

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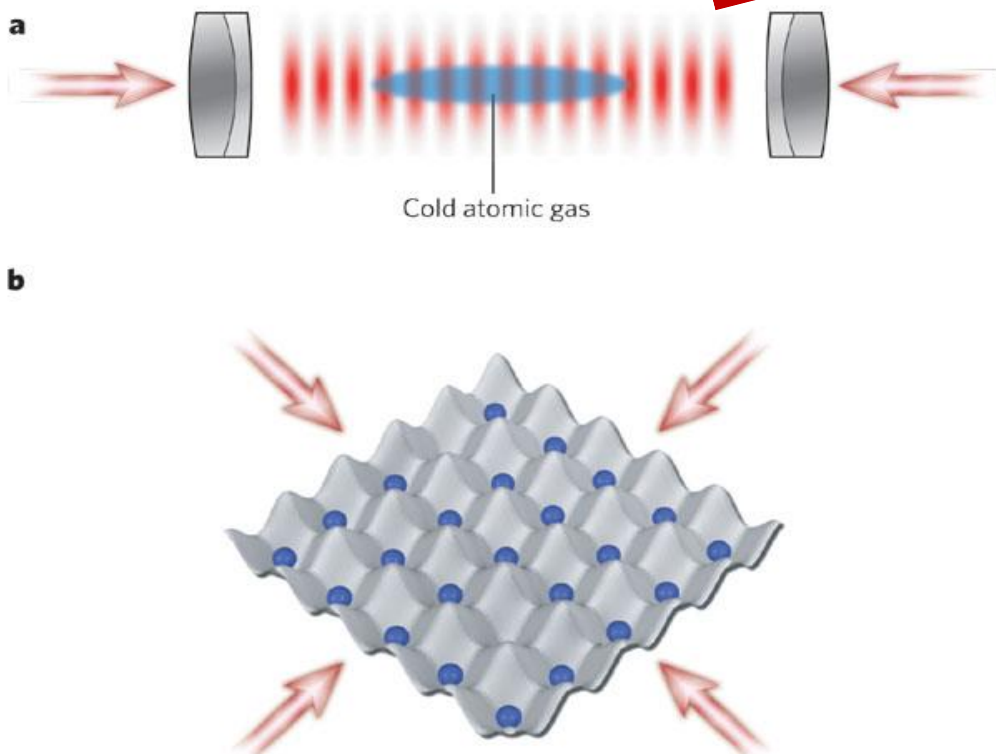
Nonlinear optics

BEC

The Gross-Pitevskii equation:

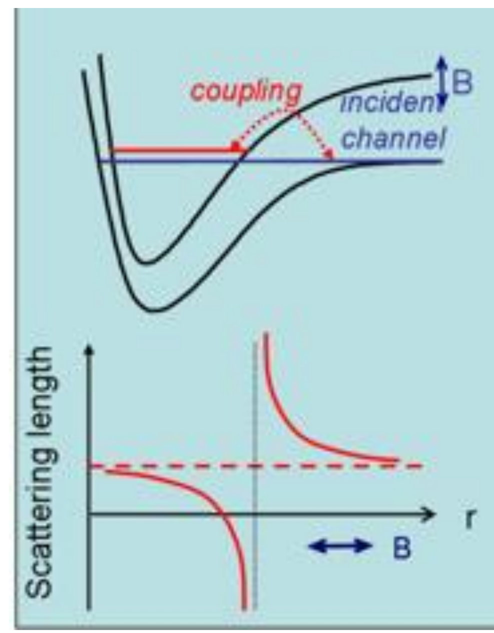
$$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+V_{OL}(\vec{r})+\frac{4\pi\hbar^2 N a_{SL}}{m}\left|\Psi(\vec{r},t)\right|^2\right]\Psi(\vec{r},t)$$

Optical Lattice



Nonlinearity

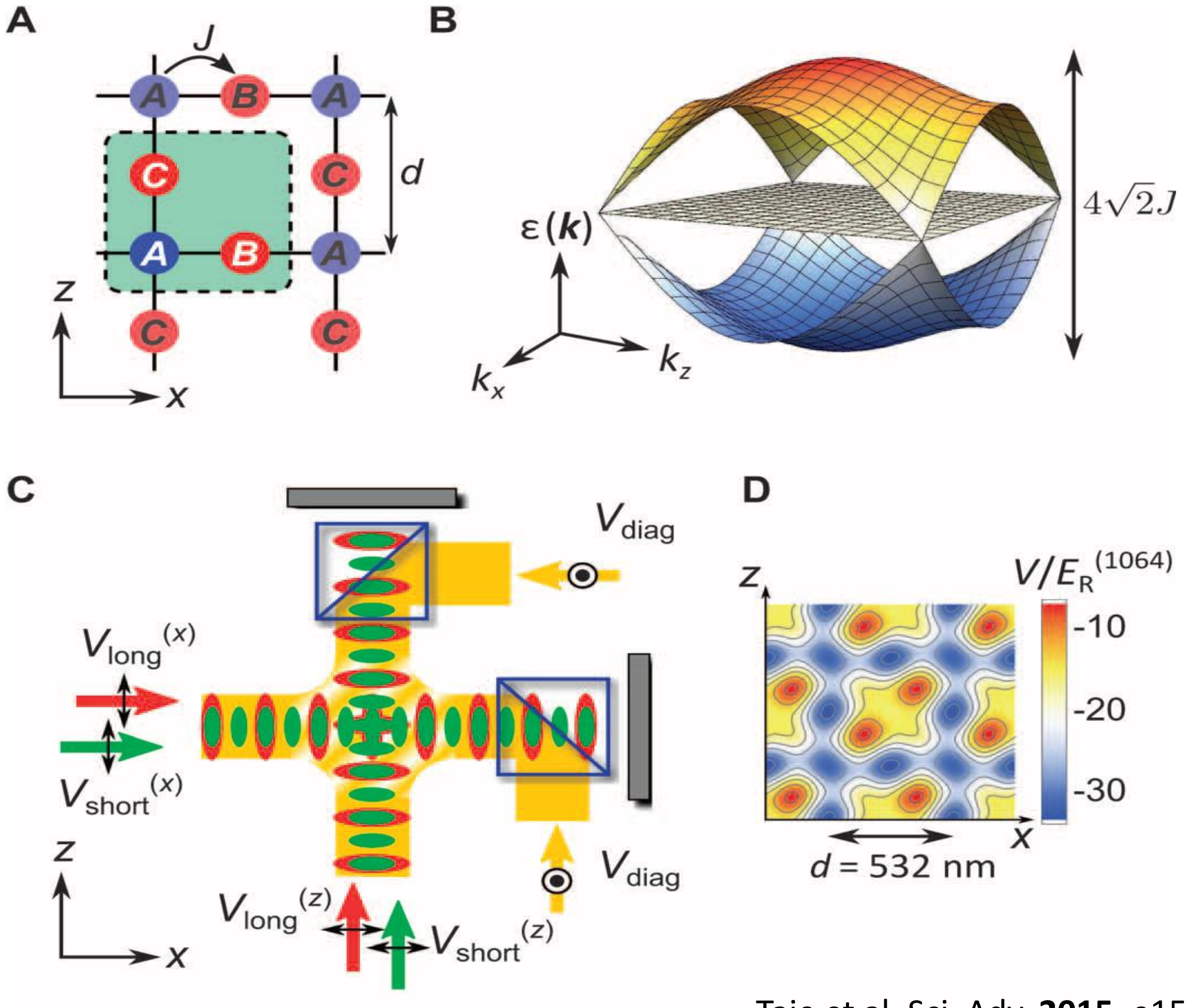
$$a_{SL}=a\left(1+\frac{\Delta_r}{B-B_0}\right)$$



I. Bloch, *Nature* **453** (2008)

FB systems in BEC

$$V(x,z) = -V_{long}\cos^2(k_L x) - V_{long}\cos^2(k_L z) - V_{short}\cos^2(2k_L x + \phi_x) - V_{short}\cos^2(2k_L z + \phi_z) - V_{diag}\cos^2(k_L(x-z) + \psi)$$



Taie et al. Sci. Adv. **2015**, e1500854 (2015)