SCT-17

Quantum analogue of the of Kelvin-Helmholtz instability on the free surface of He-II induced by the relative motion of superfluid and normal components of the liquid along the surface

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Properties of liquid hydrogen, helium and water

	H_2 T=15 K;	^{4}He, 4.2 K;	He-II, 1.8 K;	H_20 300K
Density **ρ, g/cm^3**	0.076;	0.14;	0.14;	1.0
Surface tension **б, dyn/cm**	2.7;	0.12;	0.32;	77
Capillary length $\lambda_c=2\pi(б/\rho g)^{1/2}$, **cm**	1.18	0.18	0.30	1.7
Nonlinearity coefficient $(4б/\rho^3)^{1/4}$, **cm$^{9/4}$/g$^{1/2}$ s$^{1/2}$**	8.9	2.5	3.19	3.0
Kinematic viscosity **ν, cm^2/s**	0.0026	0.00026	0.000089	0.01

Faraday instability of superfluid surface. ***PHYSICAL REVIEW E 76, 046305 _2007_Haruka Abe, Tetsuto Ueda, Michihiro Morikawa, Yu Saitoh, Ryuji Nomura, and Yuichi Okuda**,* *W*e observed that Faraday waves are parametrically generated on a free surface of superfluid 4He when a sample cell is vibted vertically. Standing-wave patterns appear on the surface, and their frequencies are one-half the driving frequency. **We observed clear threshold amplitudes of the vibration for the instability**. **The difference in the threshold between the superfluid and the normal fluid is explained by a wall damping**.

Introduction to quantum turbulence. ***Carlo F. Barenghi, Ladislav Skrbek, and Katepalli R. Sreenivasan PNAS March 25, 2014, vol. 111 no. Supplid 1 4647–4652*** Quantum fluids differ from ordinary fluids in three respects: (i) they exhibit two-flow turbulence behavior at nonzero temperature or in the presence of impurities, (ii) they can flow freely, without the dissipative effect of viscous forces, and (iii) their local rotation is constrained to discrete vortex lines of known strength (unlike the eddies in ordinary fluids, which are continuous and can have arbitrary size, shape, and strength). **Superfluidity and quantized vorticity are extraordinary manifestations of quantum mechanics at macroscopic-length scales.**

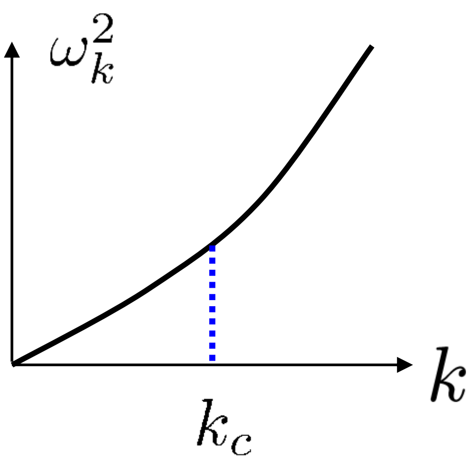
Superfluid Boundary Layer, ***G.W. Stagg, N. G. Parker, and C. F. Barenghi, PRL 118, 135301 (2017)*** We model the superfluid flow of liquid helium over the rough surface of a wire (used to generate turbulence) profiled by atomic force microscopy. Numerical simulations of the Gross-Pitaevskii equation reveal that the sharpest features in the surface induce vortex nucleation both intrinsically (due to the raised local fluid velocity) and extrinsically (providing pinning sites to vortex lines aligned with the flow). Vortex interactions and reconnections contribute to form a dense turbulent layer of vortices with a nonclassical average velocity profile which continually sheds small vortex rings into the bulk. We characterize this layer for various imposed flows. **Vortex interactions and reconnections contribute to form a dense turbulent layer of vortices with a nonclassical average velocity profile which continually sheds small vortex rings into the bulk. We characterize this layer for various imposed flows. As boundary layers conventionally arise from viscous forces, this result opens up new insight into the nature of superflows**

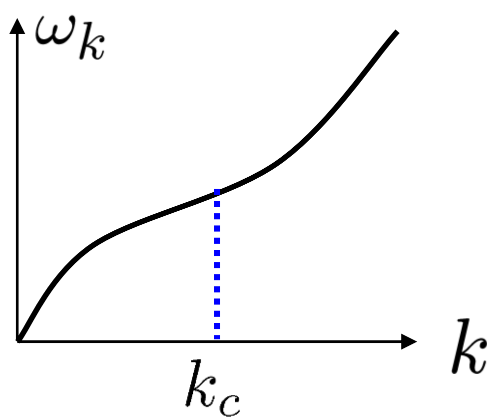
Linear waves on the free surface of liquid

Dispersion law for linear capillary-gravity waves (waves of small amplitude)

$$\omega_k^2 = gk + \frac{\alpha}{\rho}k^3$$

$$k_c = \frac{2\pi}{\lambda_c}$$





In accordance with the previous table,
for water at T=20 C one has: k_c=3.6 cm^{-1},
for liquid hydrogen at T=15 K $\boldsymbol{k_c}$**=5.32** cm^{-1}
and ***for He-II at T=1.8K*** $\boldsymbol{k_c}$**≈ 11 cm**$^{-1}$

g is the free fall acceleration
α is the surface tension
ρ Is the density of liquid

In case of long (gravity) waves, $k<k_c$, ω << 10 Hz in He-II, the dispersion relation reads

$$\omega_k = \sqrt{gk}$$

For short (capillary) waves, $k>k_c$, ω >> 30 Hz the dispersion relation is the following

$$\omega_k = (\alpha/\rho)^{1/2}\,k^{3/2}$$

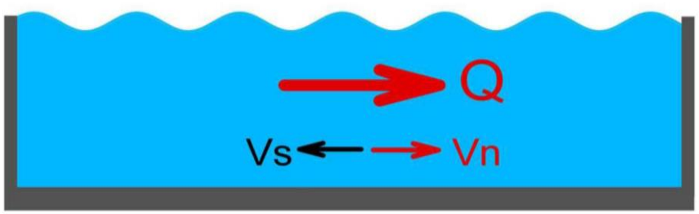
IINTRODUCTION: A dynamical instability on the surface of superfluid He-II induced by the relative motion (counterflow) of normal and superfluid components under the action of a stationary heat flow within the liquid. The two-flow instability in superfluid He-II due to relative motion of the superfluid and normal components in bulk *(the vapor over a liquid surface remains motionless*) could be perhaps one of the simplest and clearest cases.

Theory: S.E. Korshunov
Europhys Lett 16:673, 1991;
JETP Lett 75:423, 2002

$v_n = (Q/\Sigma)/(\rho ST)$

$$\Delta p = 1/2(\rho_n v_n^2 + \rho_s v_s^2)$$

$$|w| = |v_n| + |v_s| = |v_n|\rho/\rho_s$$



The nondissipative two-fluid description : the viscosity coefficient is zero $\eta = 0$ and the dispersion relation looks as

$$\omega^2 = gk + (\sigma/\rho)k^3 - (wk)^2(\rho_n\rho_s/\rho^2)$$

The corrugation instability of the free surface starts to develop on a critical wave vector **$k_c = (\rho g/\sigma)^{1/2}$,** where **$k_c \approx 11$ cm^{-1}** at T = 1.8 K is weakly dependent on temperature T.

$$w_{c0}^2 = 2(\rho^3 g\sigma)^{1/2}/(\rho_n\rho_s)$$

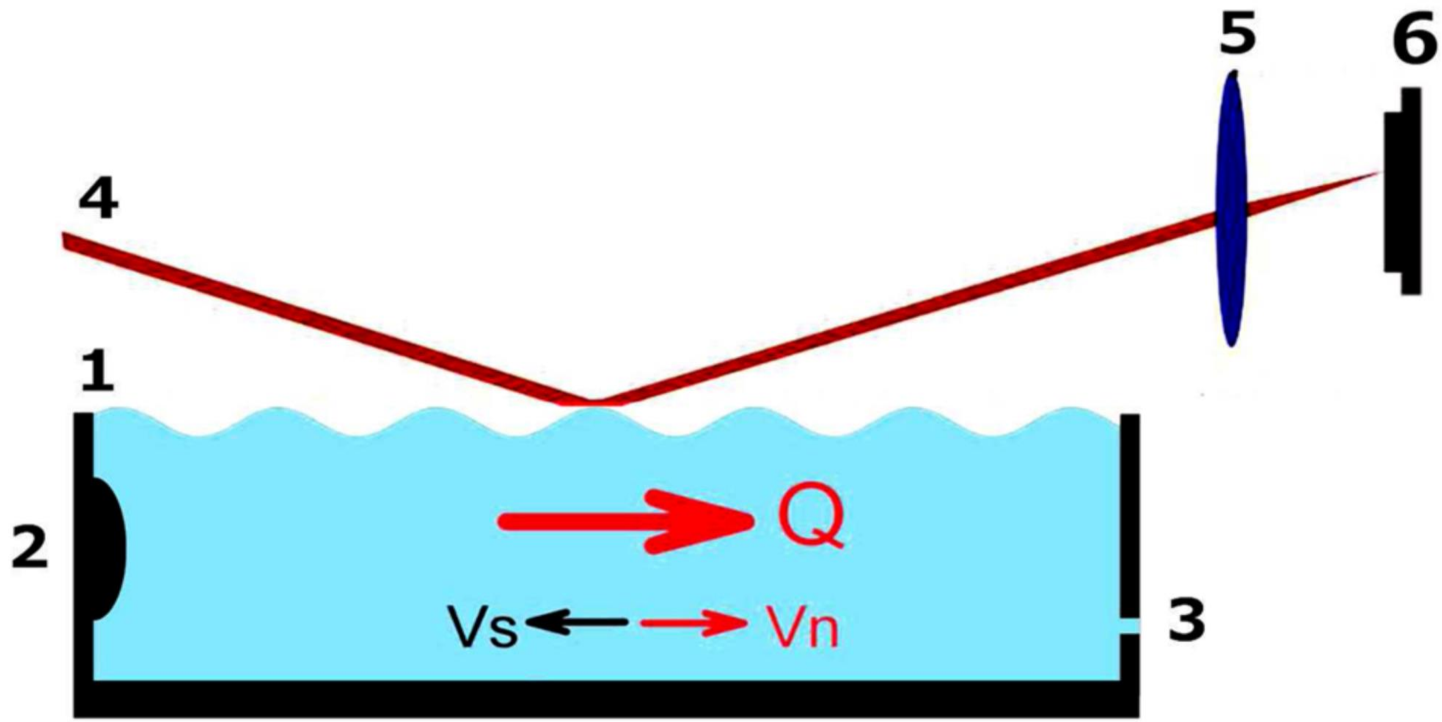
For any finite $\eta > 0$ the dispersion relations should be modified (JETP Lett, 2002) and the reconsidered value of the critical velocity is given as

$$w_c^2 = 2(\rho g/\sigma)^{1/2}/\rho_s = w_{c0}^2(\rho_n/\rho)$$

and the value of the threshold heat flux density (Q/Σ)thr

$$(Q/\Sigma)_{thr} \geq ST(2k_c\sigma\rho_s)$$

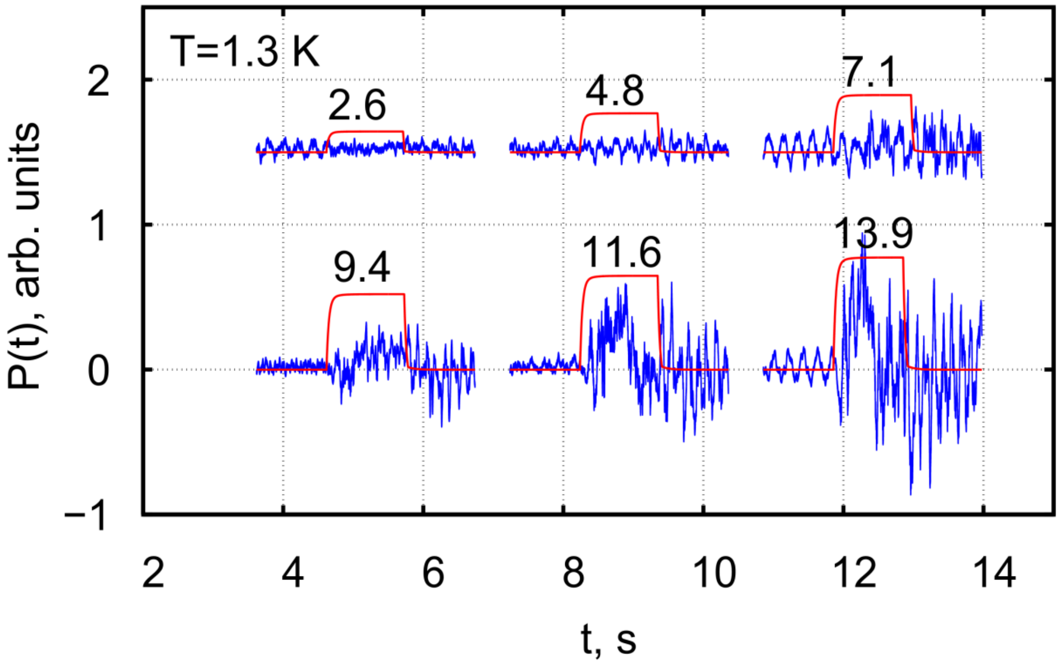
Scheme of the experiment: 1- the rectangular container; 2- the resistive heater ; 3 - holes in the lateral walls (used in the main experiments, only); 4 - the laser beam; 5 - the lens, and 6 - the photo detector



The container 1 of inner sizes of 30×24 mm and the depth of 5 mm was made of the material with low heat conductivity (plexiglass), and it was mounted inside the optical cell filled with He-II and thermally connected the with the helium vessel of the optical cryostat. The heat flux in the container was radiated by the thin metal film 2 with the surface $\Sigma = 1.2\ cm^2$ mounted on a lateral wall of the container and was transferred to the outer He-II bath through the holes 3 on the walls of container

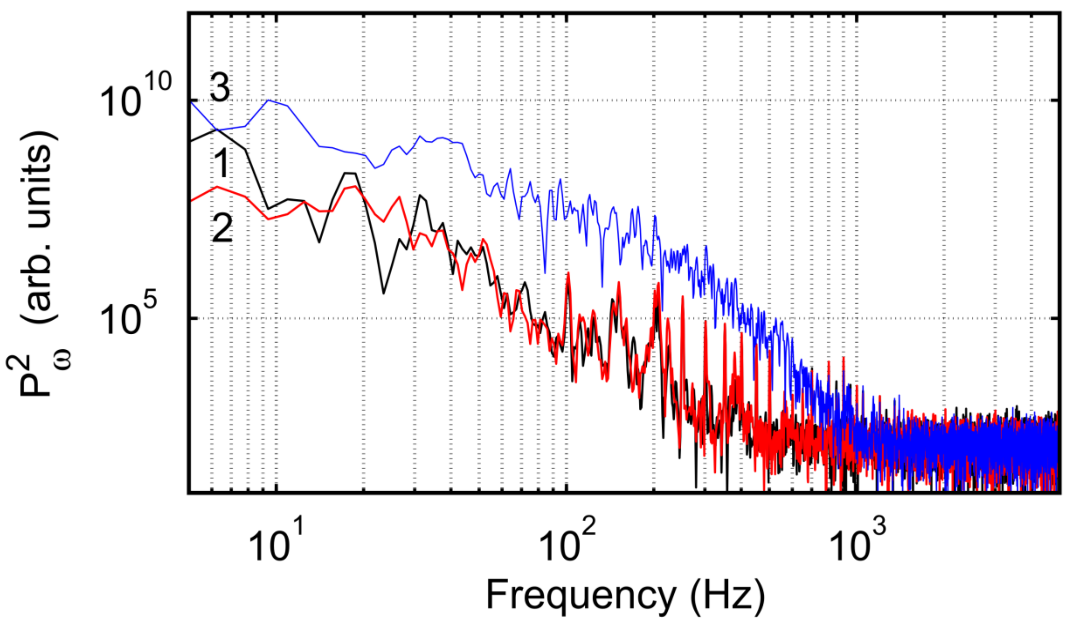
Experiment. The surface oscillations were monitored by recording tte power variation of the reflected laser beam from the liquid surface. The beam was incident on the surface at a small grazing angle. The maximum wave amplitude at the liquid surface before transition to a corrugated state did not exceed 0.2 mm typically. The reflected beam was focused onto the photodetector by the outer lens. The variable component of the laser beam power **P(t)** was recorded by a computer. To clear the frequency distribution in the spectra of surface oscillations **P(t)** we analyzed the frequency spectrum P_ω obtained by the time Fourier-transform of the recorded **P(t)-**dependence (P_ω denotes the absolute value of the Fourier transform). The power variation of the thin laser beam is proportional to the deviation angle, $P(t) \approx \varphi(t)$, where $\varphi = \eta / \lambda$ is the ratio of the wave amplitude η to its length λ.
The correlation function of surface elevation I_ω in the low-frequency limit (**gravitational waves), the frequency $f \ll 30$ Hz**, the narrow beam approximation) can be written in the frequency representation as follows: $I_\omega = |\eta_\omega|^2 \sim P^2_\omega\, \omega^{-4/3}$,
where P^2_ω is the distribution in frequency of squares of the Fourier component for the experimentally measured time dependence of the power of the reflected beam **P(t).**
And in the high-frequency limit (**$f \gg 30$ Hz, capillary waves**) $I_\omega \sim P^2_\omega$.
The theory cannot predict the spectrum of spatial and temporal vibrations at the heat flow above the threshold, though in [3] the authors have associated the surface vibrations with generation of the acoustic excitations. In accordance with the theoretical predictions the growth time of instability should strongly decrease with raising the heat flow above the threshold, and it diverges as the heat flow approaches the threshold from above. This greatly limited the accuracy of the threshold definition in experiments with rectangular heat pulses

EXPERIMENT: Rectangular heat pulses



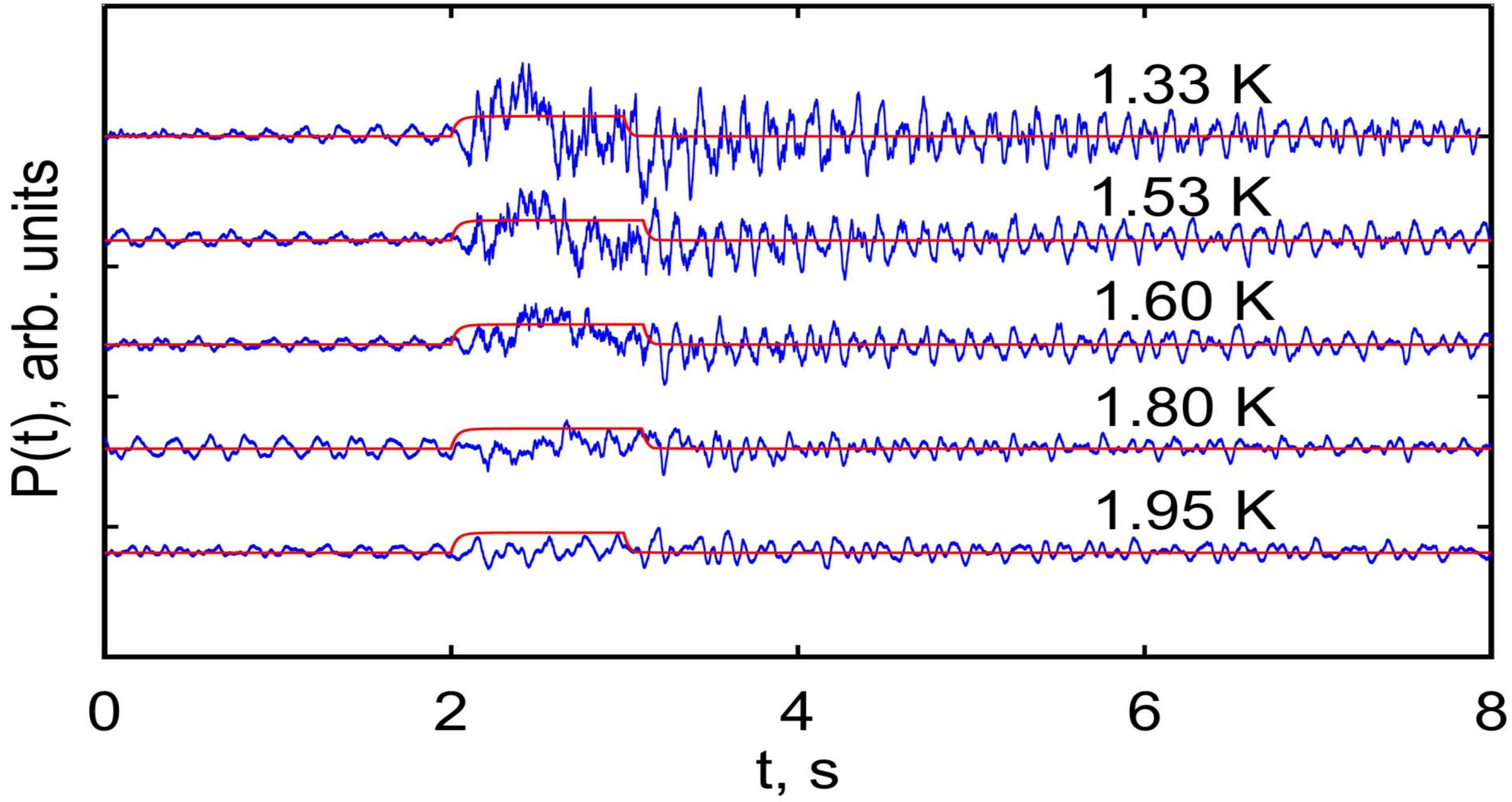
Evolution of the shape of **surface vibrations P(t)** on heating the resistor in He-II by the rectangular pulses (shown by straight lines). The bath temperature T = 1.3 K. The pulse duration is t = 1 s. The figures above the pulses indicate the pulse amplitudes **U**

$$Q/\Sigma = U^2/(R\Sigma) \approx 3\cdot 10^{-3}\, U^2 \text{ W/cm}^2$$

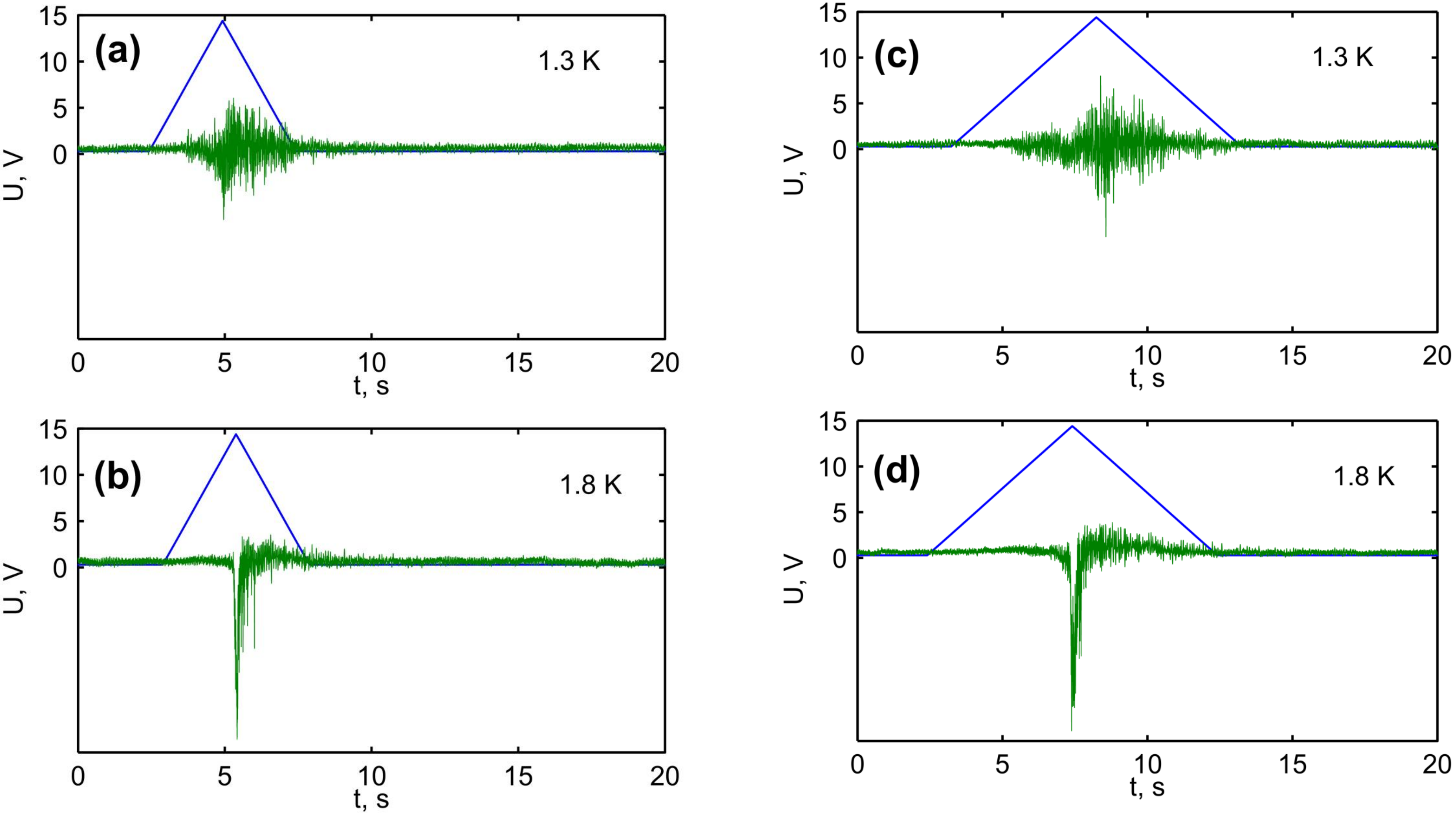


The frequency distribution of the power of the **surface elevations P^2_ω**
The curve **1(dark)** corresponds to the background vibrations at zero **U =0**, recorded before switching on the pulse.
The curves **2(red)** and **3(blue)** show evolution of the frequency distribution **P^2_ω** with increasing the pulse amplitude from **U = 2.6 to 11.6 V** , accordingly.
The bath temperature is T = 1.3 K

Evolution of the shape and amplitudes of the surface vibrations **P(t)** with increasing the temperature of He-II. The pulse duration is **t =1 s**, the pulse amplitude **U = 11.6 V**

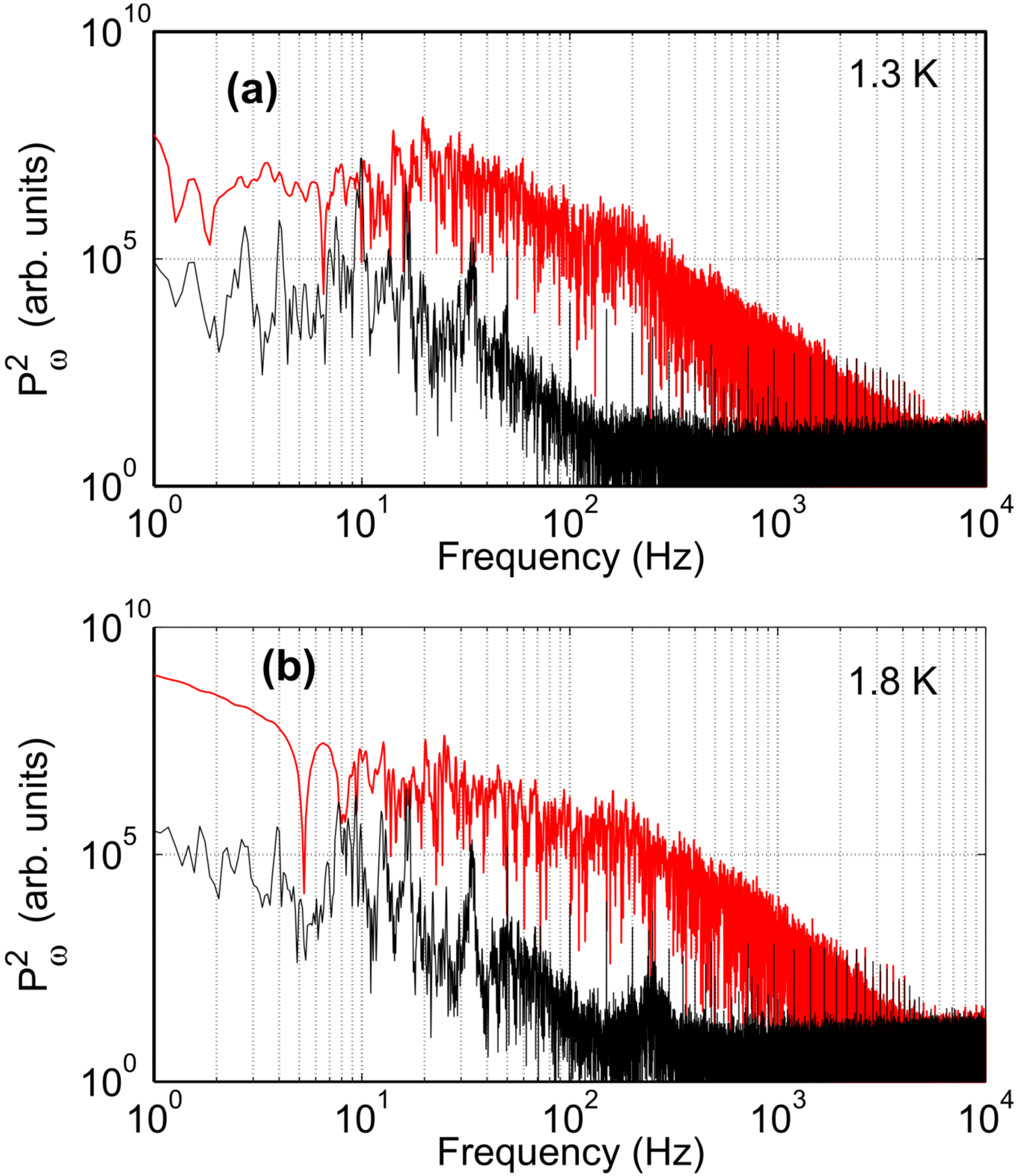


Triangular voltage pulses



Changing in the shape and amplitude of the surface oscillations over time under the action of a quasi-stationary heat flow at temperatures T=1.3 (a, c) and 1.8 K (b, d) . The time duration of the triangular voltage pulses is 5 s (left frames) and 10 s (right frames). The pulse amplitude was U=14 V.

Triangular voltage pulses

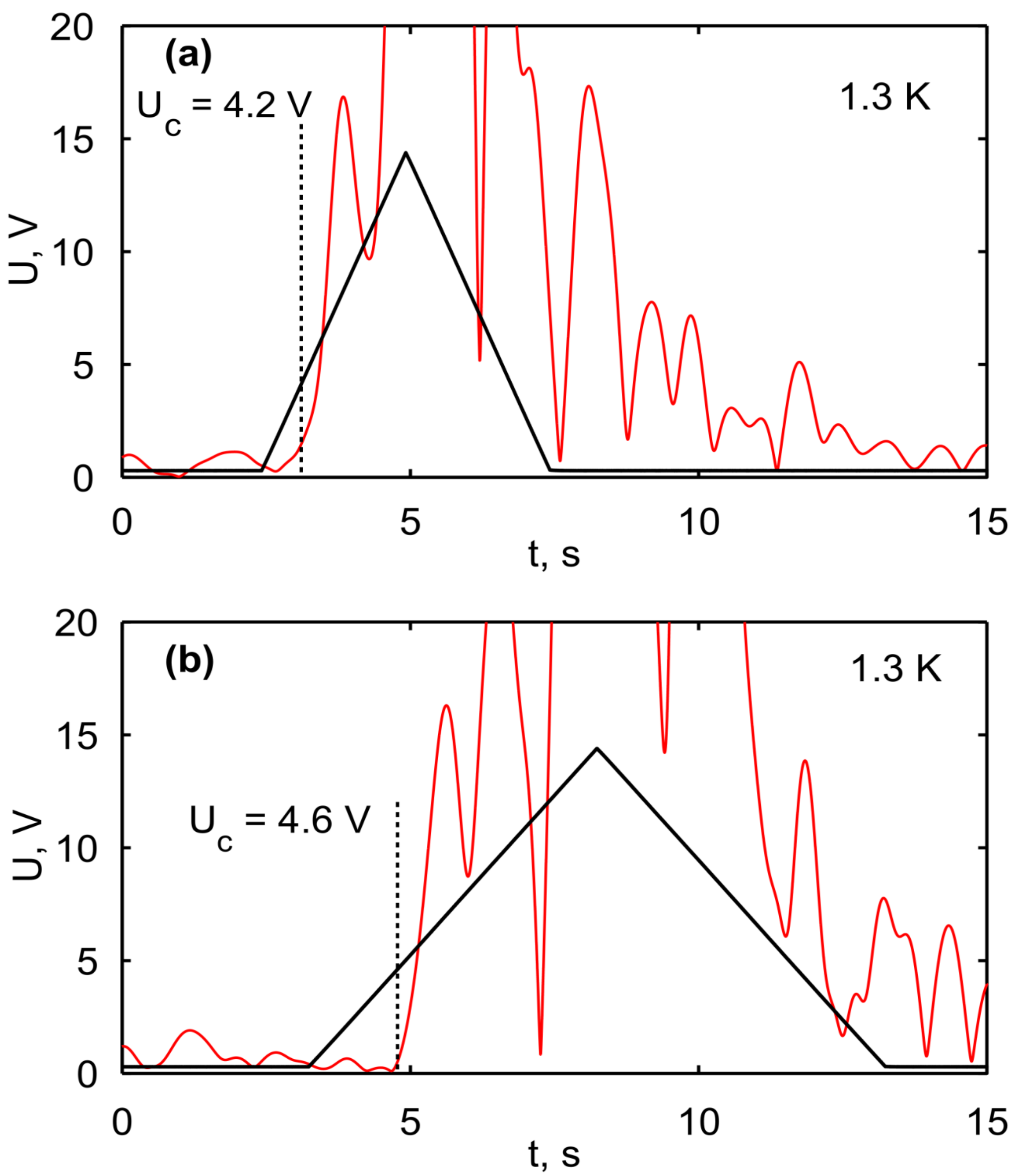


Frequency distribution of the power spectrum $\mathbf{P^2_\omega}$ of the surface oscillations generated under the action of 10 s triangular pulses at T = 1.3 and 1.8 K (pulse amplitude **U = 14 V**).

Lower dark curves correspond to the background noise oscillations registered during 10 s time domain just before switching on the electrical pulse.

The upper red curves show the evolution of the frequency distribution under the action of the heat flux.

Estimation of the threshold voltage U_{thr}



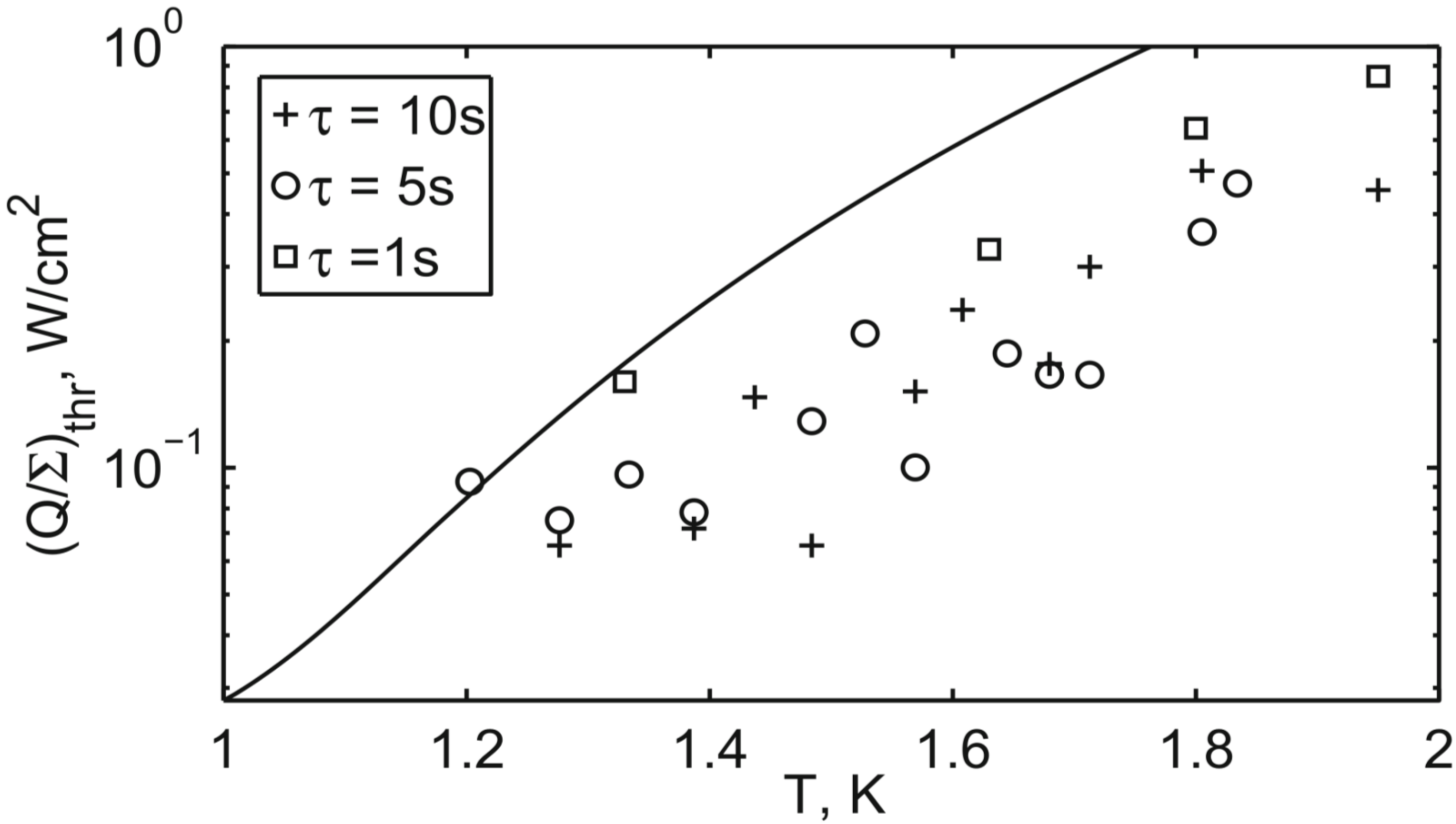
An example of estimation of the threshold voltage U_{thr} corresponding to an initial moment of generation of an instability at the surface of He-II at T = 1.3 K for two triangular pulses of different duration 5 s (a) and 10 s (b).

Triangular pulses shown by the straight lines describe the time evolution of the voltage applied to the heater;

oscillating red curves show changes over time of the amplitude of the wave at the selected frequency $f=\omega/2\pi$ = 20 Hz in the frequency spectrum P(t) shown in previous slide.

Vertical dotted lines show the time, when the signal envelope starts to grow, attributed to the moment of an instability generation.

The threshold heat flux density



Temperature dependence of the threshold heat flux density **(Q/)**$_{\mathbf{thr}}$. Open circles and crosses correspond to the estimations made for triangular pulses of duration of 5 and 10 s, and the open squares correspond to rectangular pulses. The solid curve demonstrates the results of the numerical estimations of **(Q/Σ)**$_{\mathbf{thr}}$ calculated in accordance with the results of the theoretical consideration [2]: $\boldsymbol{(Q/\Sigma)_{thr} \geq ST\ (2k_c\sigma\rho_s\ )}$

Short discussion and conclusions

The *temperature dependence* of $(Q/\Sigma)_{thr}$ predicted by the Korshunov's theory agrees fairly well with our experimental data, though *the results of numerical estimations* shown by points can differ up to an order of magnitude with the theory. For the correct comparison of the experimental data shown by points with the predictions of the theory (solid curve) we need:

first, a better knowledge of the velocity profile inside the liquid, and the influence of this profile on the development of the surface corrugation at large heat fluxes;

second, it is known that the growth time of instability at heat fluxes above the threshold should shorten quickly when the flux is increased above the threshold, but in the vicinity of the critical point it should be extremely large, thus the experimental values of $(Q/)_{thr}$ might be even overestimated;

third, in a number of recent experiments the authors have studied motion of a cluster of excited He molecules in a long rectangular channel of the cross-section ≈ 0.9 cm^2. The applied heat fluxes were varied between 0.16 and 1.0 W/cm^2, far above the critical value, above 0.050 W/cm^2 sufficient for creation of a quantum turbulence in bulk of He-II according to Vinen. It is likely that, in the small heat flux regime $Q/\leq 50$ mW/cm^2, both the superflow and the normal-fluid flow are laminar, and a profile of the normal fluid flow in a long channel is close to a Poiseuille (parabolic), but above a certain critical heat flux above 50 mW/cm^2, the superflow is known to become dissipative (quantum turbulence). A mutual friction force between the two fluids arises through the interaction between quantized vortices and the normal fluid. And with further increasing the heat flux *the normal fluid may also become turbulent*, mutual friction has been shown theoretically to induce instability in the laminar flow of the normal fluid. In those cases *both the two fluids are not forced to have any relative motion and behave like a single classical fluid, exhibiting a Kolmogorov energy spectrum.* Simultaneous turbulence in both fluids in a counterflow must be different, and it would be a type of turbulence that is new to physics. It is clear that in order for an instability on the surface to be relevant the unstable modes in the bulk of He-II must grow faster than all dissipation time-scales. **And we necessary for a further theoretical consideration of conditions necessary for development of the Kelvin-Helmholtz instability on the interface He-II – vapor induced by the heat flow within the liquid in continuation of the theory presented by Korshunov in [2]**. In addition one should take into account the strong nonlinearity of the surface waves and the waves of second sound generated in He-II on switching on by step the heater at high level of excitations, and also the background excitations at **zero Q** (**E. N. Kuznetsov, P.M. Lushnikov, [21]**).

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