



# Plasma Turbulence and Transport Barriers

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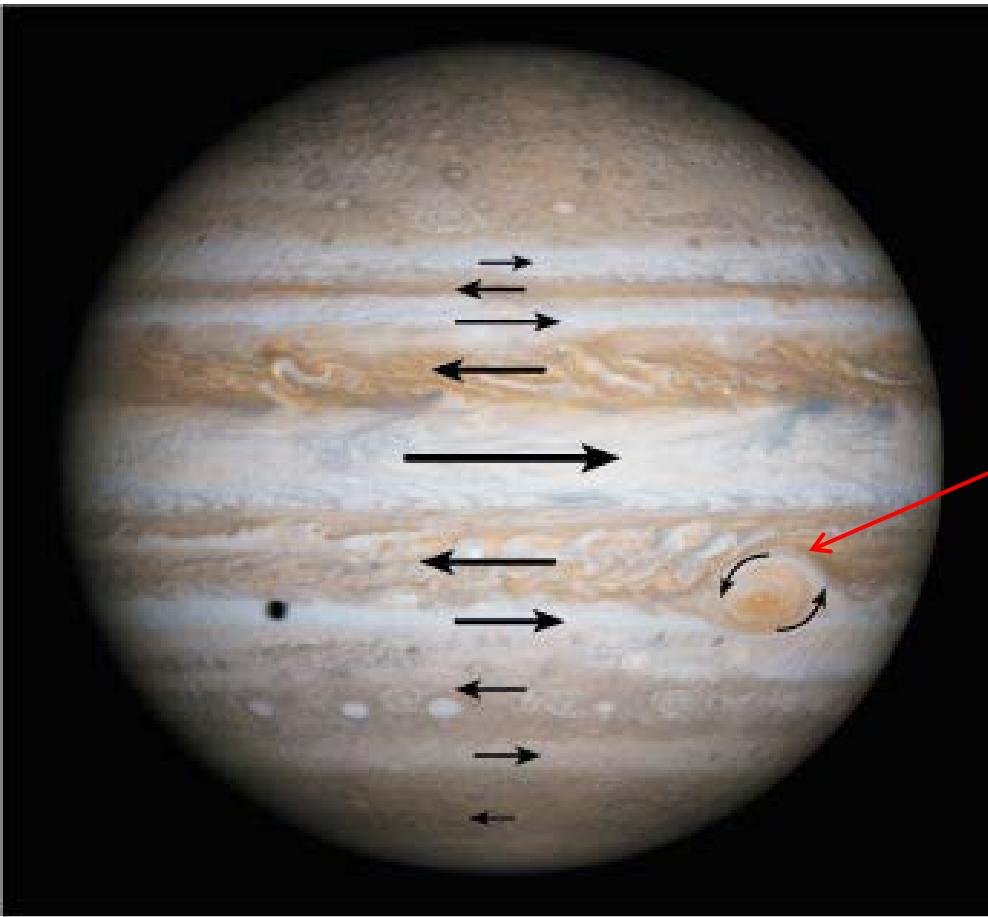
[jjra@fysik.dtu.dk](mailto:jjra@fysik.dtu.dk)

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$$
$$\int_a^b \mathcal{E} \Theta^{\sqrt{17}} + \Omega \delta e^{i\pi} =$$
$$\infty = \{2.7182818284$$
$$\chi^2 \Sigma \gg ,$$

# Jupiter zonal flow bands – classical example



Zonal flows regulate transport – transport barriers



Modeled by PV – potential vorticity - homogenization – almost – GRS anomaly

PV-staircase – piecewise constant PV

Width of the zonal bands – determined by stability – KH or Rhines scale

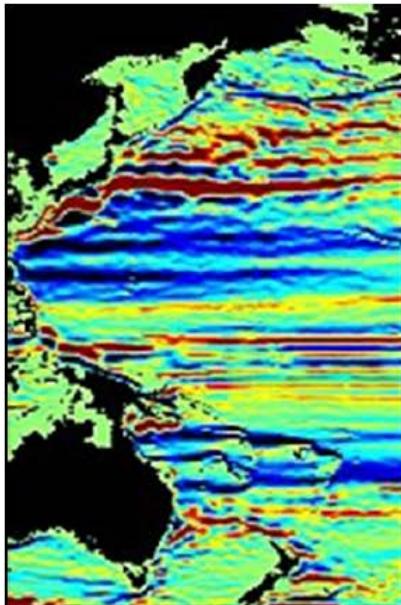
Marcus and Shetty, Phil. Trans. R. Soc. A 369, 771 (2011)

# Introduction - outline

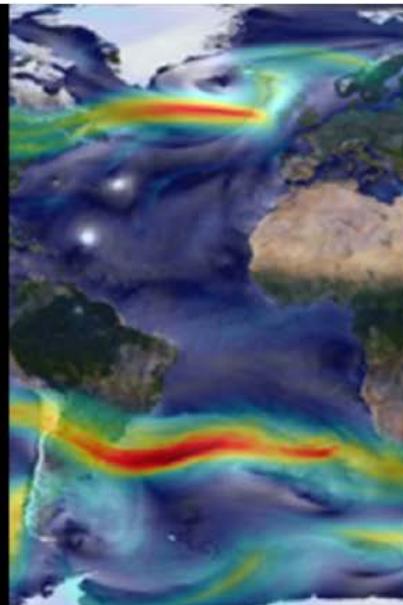
- Generation of large scale flows –zonal flows - by the rectification of small scale turbulent fluctuations is a generic feature in quasi-2D turbulent flows
- Great importance both in geophysical flows and in magnetically confined plasmas.
- The flows regulate the turbulence by suppressing the small scale structures and set up effective transport barriers.
- In magnetically confined hot plasmas the dominant cross field transport is mediated by turbulence
- Zonal flows regulate and may strongly reduce the radial turbulent transport.
- Zonal flows are instrumental in the rapid transition to an enhanced confinement state (the H-mode), with suppressed turbulent transport.
- The transition from a low (L-mode) to the high (H-mode) confinement is still not understood from first principles –
- New results on L-H transition from first principle 4-field fluid model.

# Zonal flows in geophysics and magnetized plasma

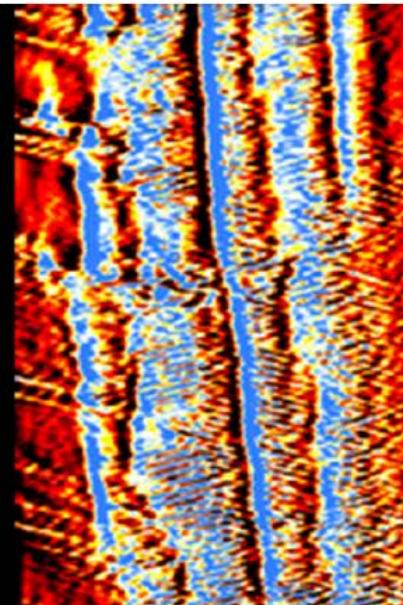
Dif Pradalier – EU-US TTF conference Leysin CH Sept 2016



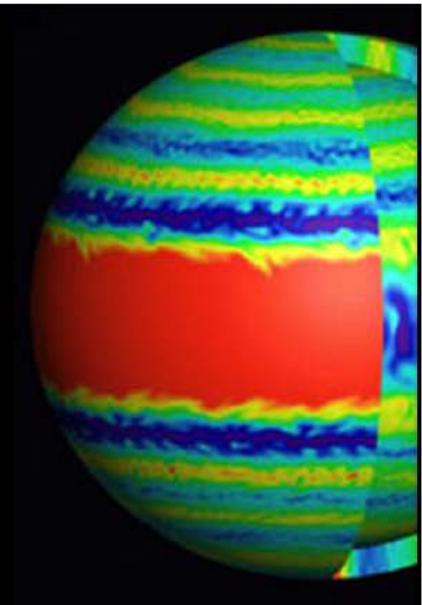
Earth ocean  
[Richards, 2006]



Earth atmosphere  
[NASA (GEOS-5) 2013]



Magn. plasma  
[Dif-Pradalier 2010]



Jupiter  
[Heimpel 2005]

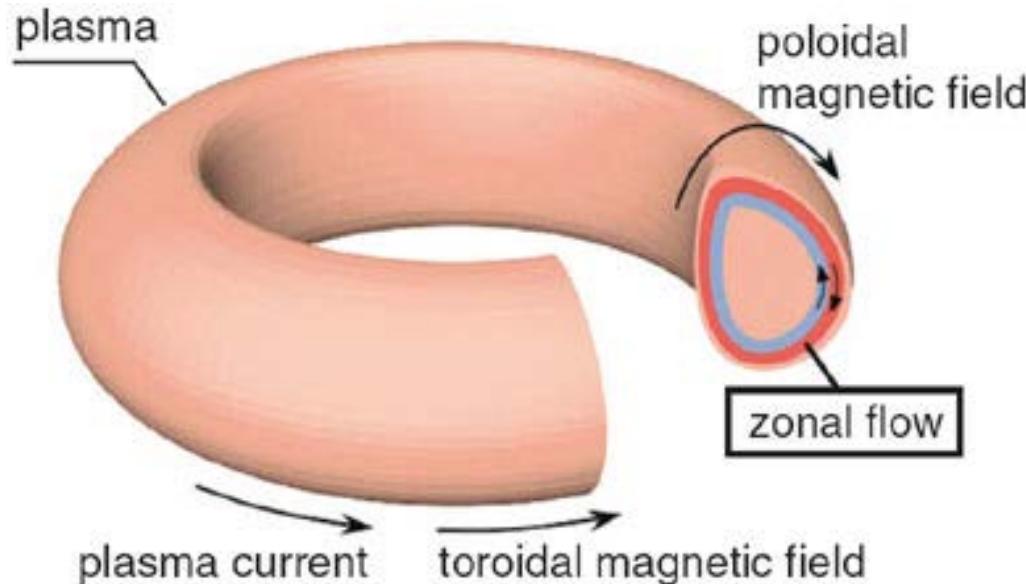
Oceans – thermocline staircases

Earth's atmosphere – and several planets – potential vorticity staircases

Magnetized plasma – PV staircases – corrugated profiles internal transport barriers

Dif Pradalier et al Nucl. Fusion 2017

# Zonal flow in magnetically confined plasma



Zonal flow :  $\mathbf{v}_{ZF} = \mathbf{E}_r \times \mathbf{B}$   
(poloidal)

$\mathbf{E}_r$  – radial electric field

Turbulent velocity  $\mathbf{v} = \mathbf{E} \times \mathbf{B} ::$

$\mathbf{v} = (u, v)$  in poloidal plane

Terminologi :

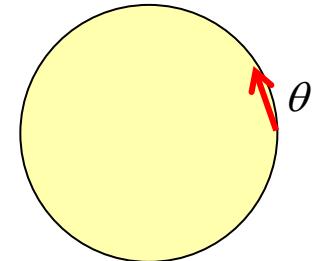
**Zonal Flows** : small scale flows driven by rectified turbulent fluctuations  
- local transport barrier Diamond et al. PPCF **47**, R35 (2005)

**Mean Flows – global poloidal flows** : large scale flows in the plasma edge – driven by radial force balance and neoclassical effects – ETB:  
edge transport barrier

# Zonal flow generation – in simple terms

- Momentum balance equation, cylindrical plasma/fluid-  $\theta$ - component

$$\partial_t v_\theta + v_r \partial_r v_\theta + v_\theta \frac{1}{r} \partial_\theta v_\theta = \mu \partial_r^2 v_\theta$$



- Reynold's decomposition

$$v_i = \bar{v}_i + \tilde{v}_i \quad \bar{v}_r = 0, \quad \langle \tilde{v}_i \rangle_\theta = 0$$

↗ Poloidal averaging

- Mean flow equation

: incompressibility  $\partial_r v_r = -\frac{1}{r} \partial_\theta v_\theta$

$$\partial_t \bar{v}_\theta = \mu \partial_r^2 \bar{v}_\theta - \partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle$$

↗ Reynolds stress – flux of momentum

# Energy equations I

- Mean zonal flow energy equation

$$\frac{1}{2} \partial_t \bar{v}_\theta^2 = \frac{\mu}{2} \partial_r^2 \bar{v}_\theta^2 - \mu (\partial_r \bar{v}_\theta)^2 - \underline{\bar{v}_\theta \partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle}$$

Transfer term

- Total energy equation

$$\frac{1}{2} \partial_t v_\theta^2 + \frac{1}{2} v_r \partial_r v_\theta^2 + \frac{1}{r} v_\theta^2 \partial_\theta v_\theta = \mu v_\theta \partial_r^2 v_\theta$$

- Fluctuating energy equation

$$\frac{1}{2} \partial_t \langle \tilde{v}_\theta^2 \rangle = -\partial_r \langle \tilde{v}_r \tilde{v}_\theta^2 \rangle - \underline{\langle \tilde{v}_r \tilde{v}_\theta \rangle \partial_r \bar{v}_\theta} + \frac{\mu}{2} \partial_r^2 \tilde{v}_\theta^2 - \mu (\partial_r \tilde{v}_\theta)^2$$

↑  
Transfer term  
Turbulence spreading

# Energy equations II

- Define:

$$\bar{K} = \frac{1}{2}\bar{v}_\theta^2$$

$$\bar{\epsilon} = \frac{\mu}{2}\partial_r^2\bar{v}_\theta^2 - \mu(\partial_r\bar{v}_\theta)^2$$

$$\bar{T} = \bar{v}_\theta \langle \tilde{v}_r \tilde{v}_\theta \rangle$$

$$P = \langle \tilde{v}_r \tilde{v}_\theta \rangle \partial_r \bar{v}_\theta$$

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$$\tilde{K} = \frac{1}{2}\langle \tilde{v}_\theta^2 \rangle$$

$$\tilde{\epsilon} = \frac{\mu}{2}\partial_r^2\tilde{v}_\theta^2 - \mu(\partial_r\tilde{v}_\theta)^2$$

$$\tilde{T} = \langle \tilde{v}_r \tilde{v}_\theta^2 \rangle$$

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- The energy equations read:

Zonal flow energy  $\rightarrow \partial_t \bar{K} = -\partial_r \bar{T} + P + \bar{\epsilon}$

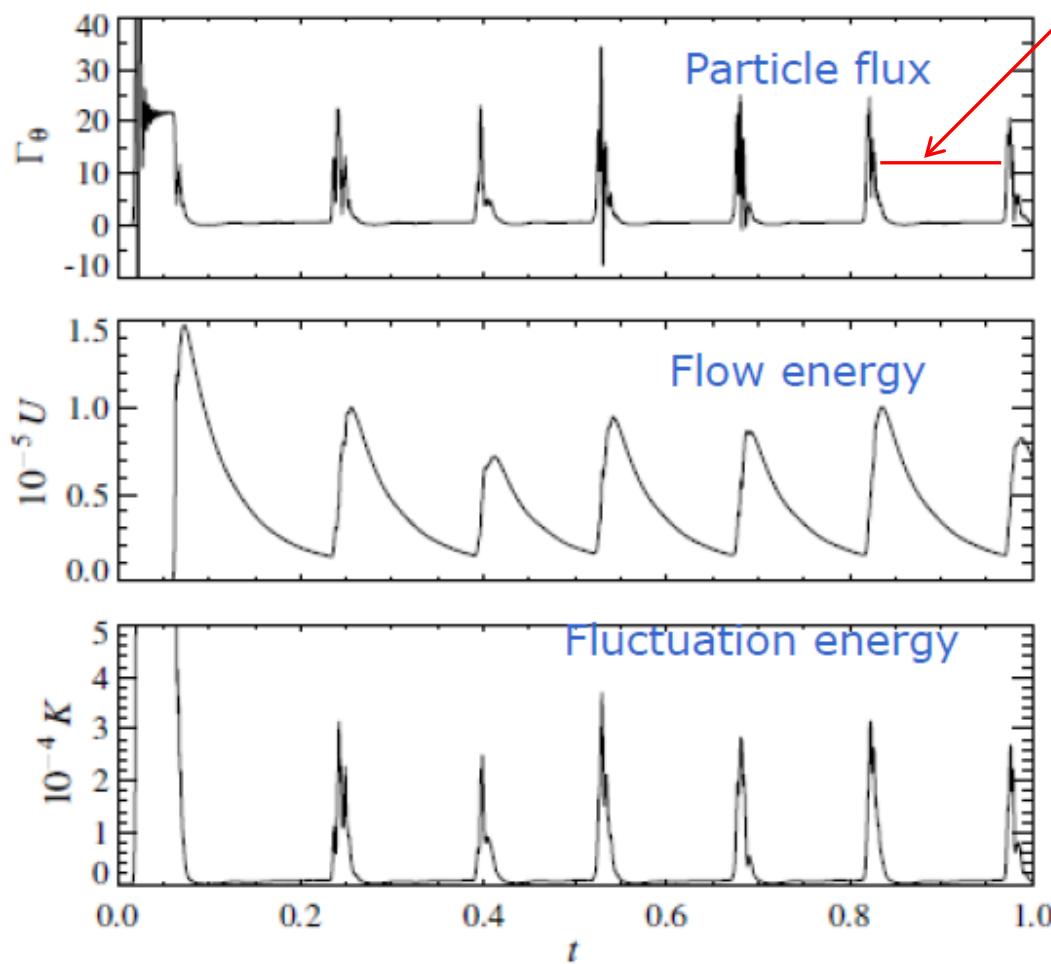
Fluctuation energy  $\rightarrow \partial_t \tilde{K} = -\partial_r \tilde{T} - P + \tilde{\epsilon}$



Adding source term for the turbulence :: predator – prey system

e.g., Kim and Diamond PRL 2003 ; Dam et al. Phys Plasma 2013

# Turbulence–flow inter play



transport “barriers”

Simulation of convection model, plasma in an inhomogeneous magnetic field.

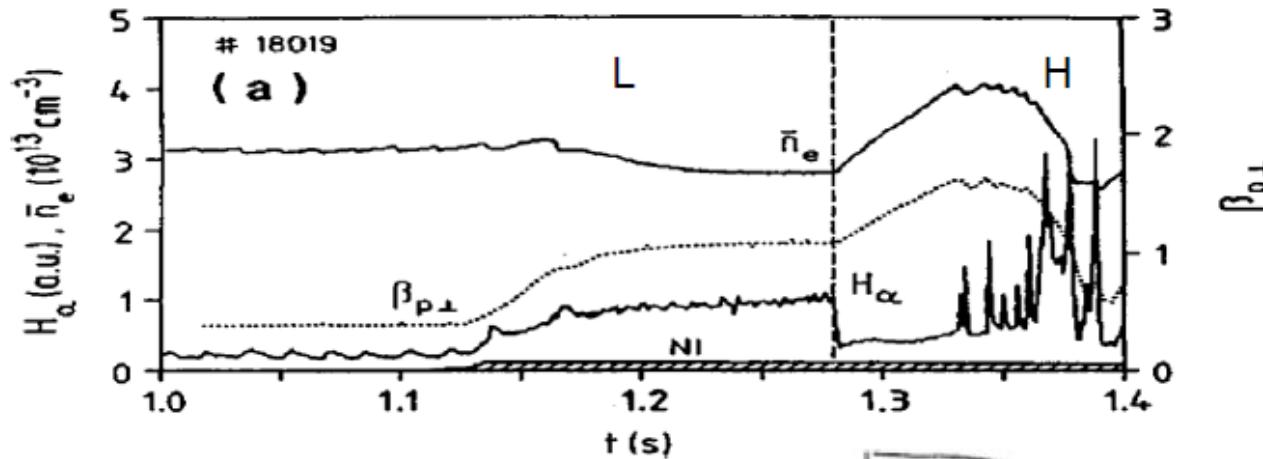
The turbulent intensity and the radial particle flux across the magnetic field is strongly modulated by the zonal flow generation.

**Typical behaviour.**

$\Gamma_\theta$  flux;  $U, K$  energy in the flow, fluctuations

Garcia and Bian PRE **68**, 047301 (2003)

# The Tokamak H-mode – result of global self-organization

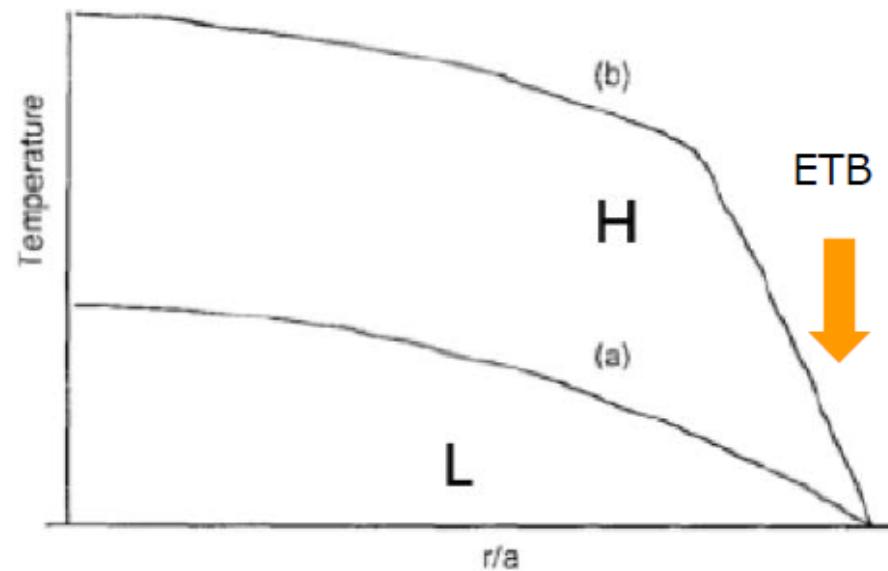


Transition – abrupt  
in response to a  
slowly changing  
parameter- energy  
input:  
“Phase transition”

Temperature (pressure)  
profiles in L- and H-mode.

H-mode essential for a viable  
operation of a fusion reactor  
– and ITER

Transport barrier (ETB) is set  
up near the edge – mediated  
by sheared poloidal flows



# L-H Transition in magnetically confined plasma



- The success of ITER – and future fusion power plants relies on controlled access to the high confinement **H-mode** –also envisaged for future fusion power plants
- **H-mode** is routinely achieved in "all" toroidal devices
- Still the **L-H transition** lacks full theoretical explanation and predictive modelling -- **turbulence flow interaction appears to play a crucial role**

## Results

- ✓ Applying a first principle fluid model
- ✓ Characteristic features of various types of **L-H transitions**: fast transition – slow transition with intermediate dithering phase
- ✓ Scaling of power threshold with density, SOL connection length
- ✓ Parameters from experiments – qualitative and quantitative agreement – medium sized tokamak (EAST/AUG) conditions

# Modelling by the HESEL model

- Energy conserving electrostatic 4-field drift fluid model
- Connects confined edge region and SOL – open field lines
- No separation of fluctuations and mean profiles
- Neo-classical transport coefficients directly from plasma parameters

$$\frac{d}{dt}n + n\mathcal{K}(\phi) - \mathcal{K}(p_e) = \nabla \cdot (nu_R) - \frac{n}{\tau_n}$$

**SOL –parallel loss terms**

$$\frac{d^0}{dt}w + \{\nabla\phi, \nabla p_i\} - \mathcal{K}(p_e + p_i) = \eta\nabla^2 w - \left[ \frac{w}{\tau_w} + \frac{enc_s}{L_{\parallel}} \left[ 1 - \exp \left( \frac{-e\langle\phi\rangle}{\langle T_e\rangle} \right) \right] \right] = \Lambda_w$$

$$\frac{3}{2}\frac{d}{dt}p_e + \frac{5}{2}p_e\mathcal{K}(\phi) - \frac{5}{2}\mathcal{K}\left(\frac{p_e^2}{n}\right) = \nabla \cdot \left( \chi_{e\perp} \nabla T_e \right) - \frac{5}{2}\nabla \cdot (p_e u_R) - u_R \cdot \nabla p_i - \frac{T_e}{\tau_{p_e}}$$

$$\frac{3}{2}\frac{d}{dt}p_i + \frac{5}{2}p_i\mathcal{K}(\phi) + \frac{5}{2}\mathcal{K}\left(\frac{p_i^2}{n}\right) - p_i\mathcal{K}(p_e + p_i) = \nabla \cdot \left( \chi_{i\perp} \nabla T_i \right) - \frac{5}{2}\nabla \cdot (p_i u_R) + u_R \cdot \nabla p_i - \frac{p_i}{\tau_{p_i}} + p_i\Lambda_w$$

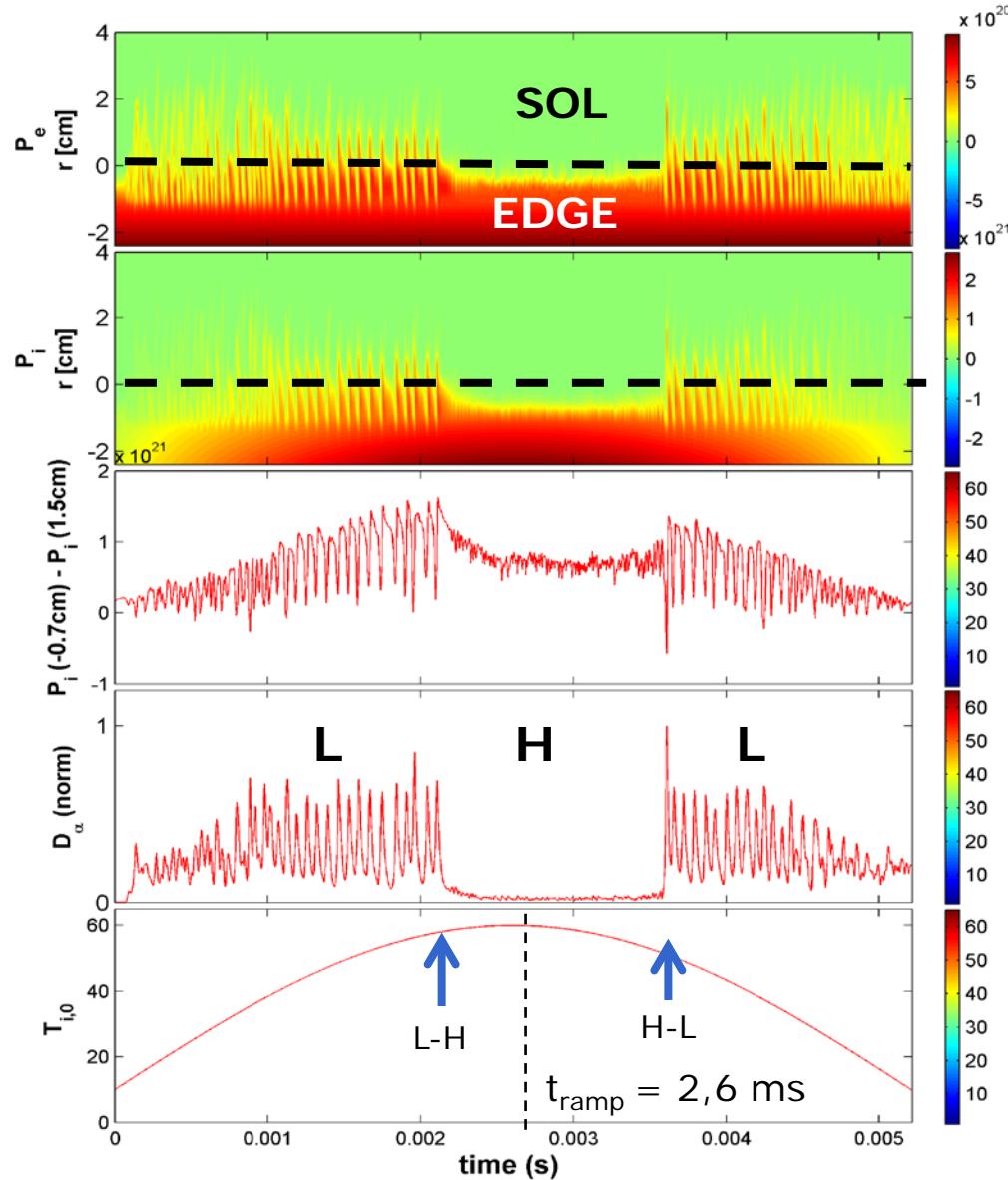
$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{B}\hat{z} \times \nabla\phi \cdot \nabla, \quad \mathcal{K} = \nabla\left(\frac{1}{B}\right) \cdot \hat{z} \times \nabla, \quad L_{\parallel} \text{ -- connection length}$$

$$w = \nabla^2\phi + \nabla^2p_i \text{ --- generalized vorticity} \quad p_i \text{ - ion pressure}$$

Essential for setting up and sustaining “mean flow”

Madsen et al PoP 2016, Nielsen et al PLA 2015, Rasmussen et al PPCF 2016

# L-H-L transition



Electron pressure profile

Ion pressure profile

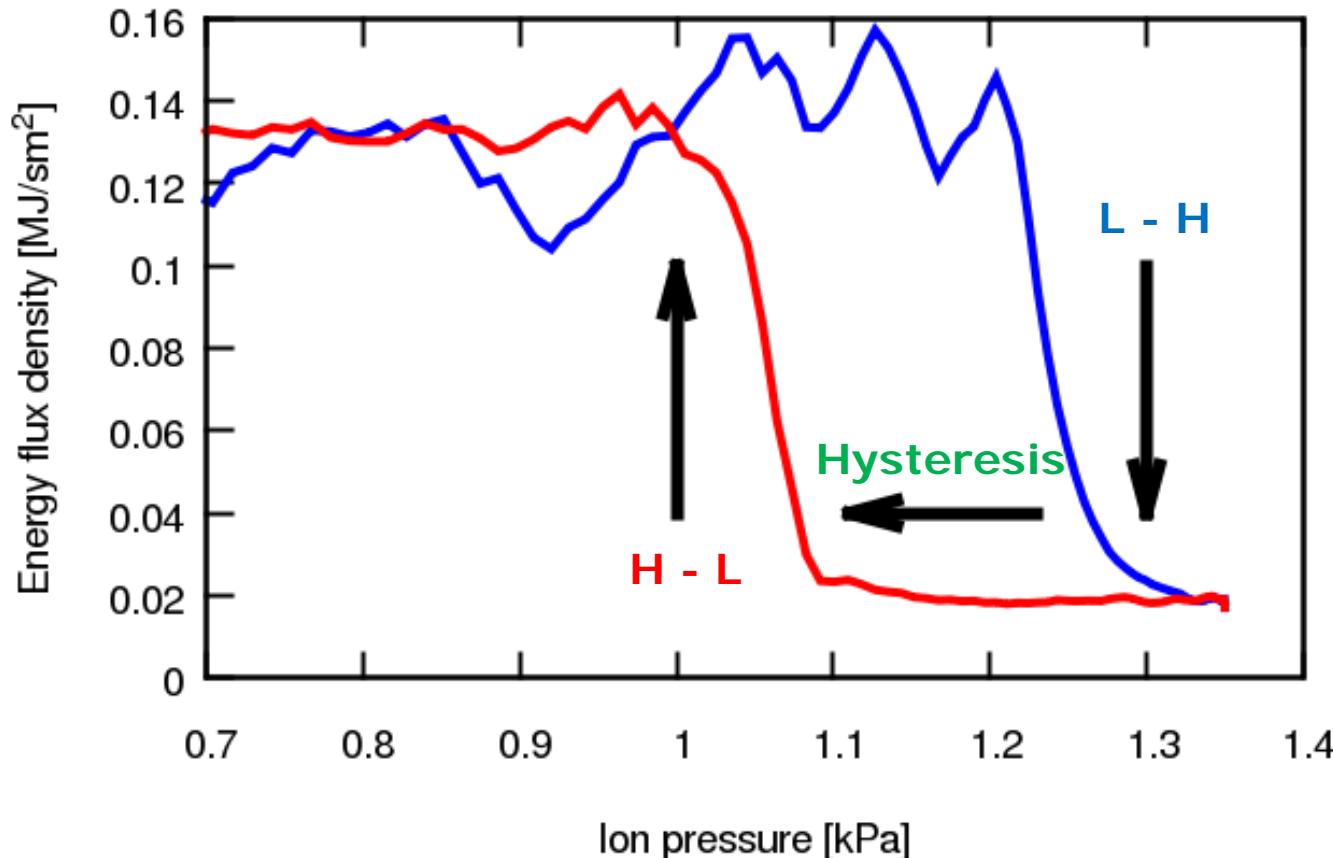
Ion pressure difference  
across LCFS (Last closed  
flux surface)

Integrated || energy flux @  
outboard mid-plane – proxy  
for  $D_\alpha$

Power input: ion temperature  
ramp-up - increase of ion  
heat flux

Rasmussen et al PPCF (2016) **58**, 014031

# Energy flux density at LCFS for L-H-L phase



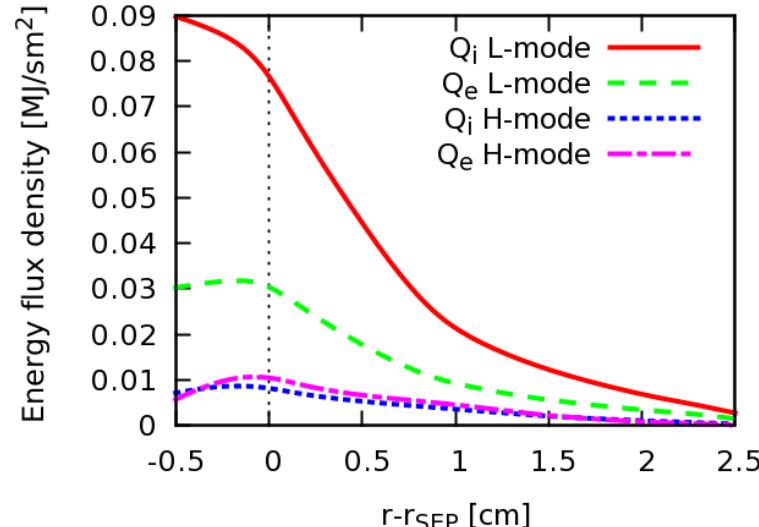
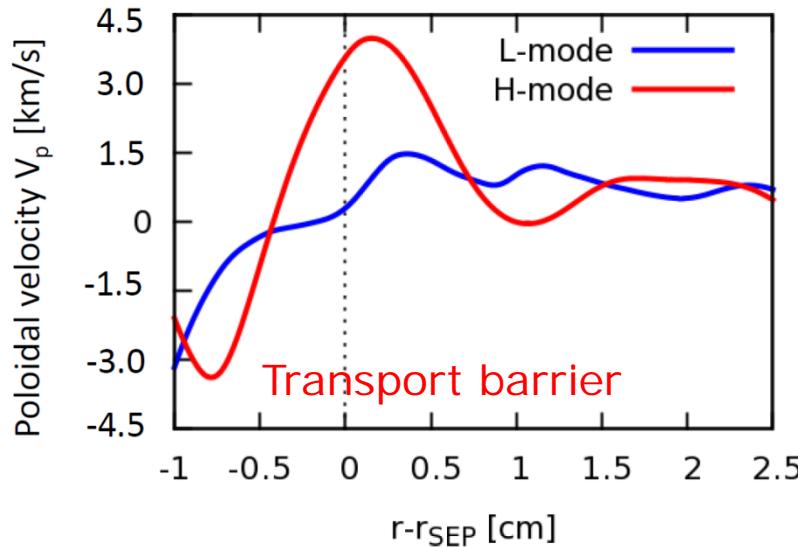
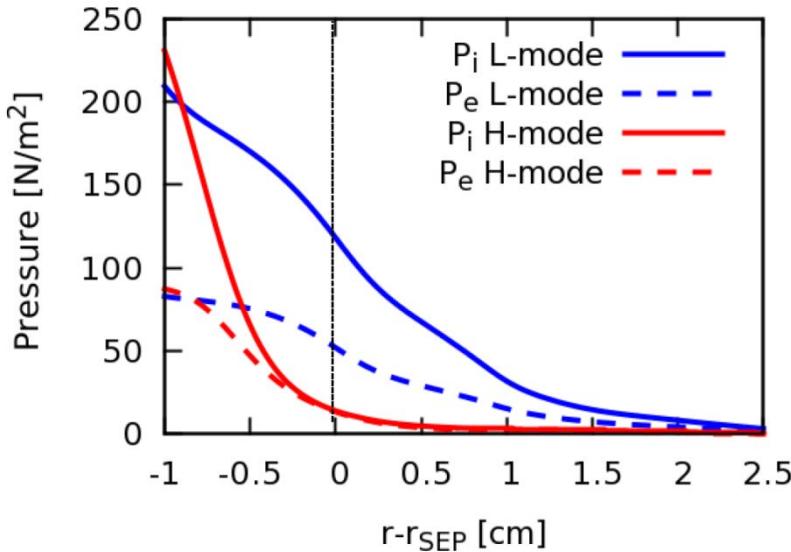
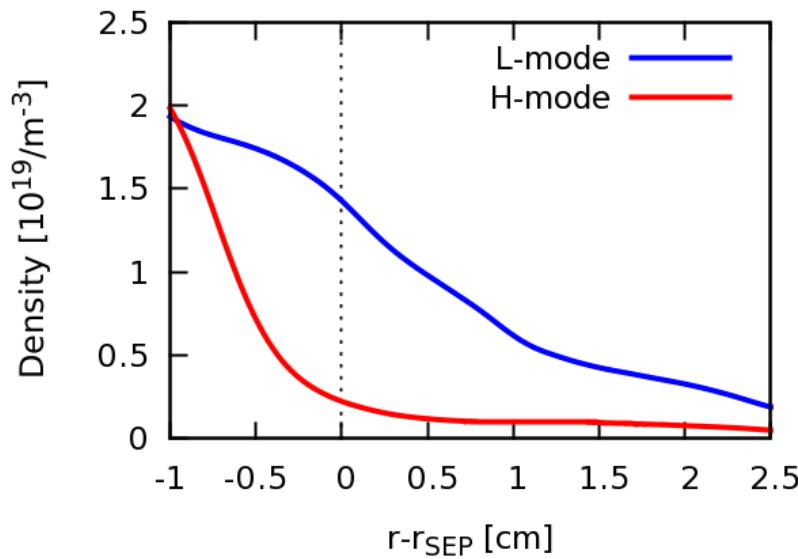
H-mode significantly decreased energy flux – improved confinement by changing ion temperature - energy flux adjust consistently

Threshold power  $P_{th} \sim 1.2$  MW – close to observed  $P_{th}$  @ EAST

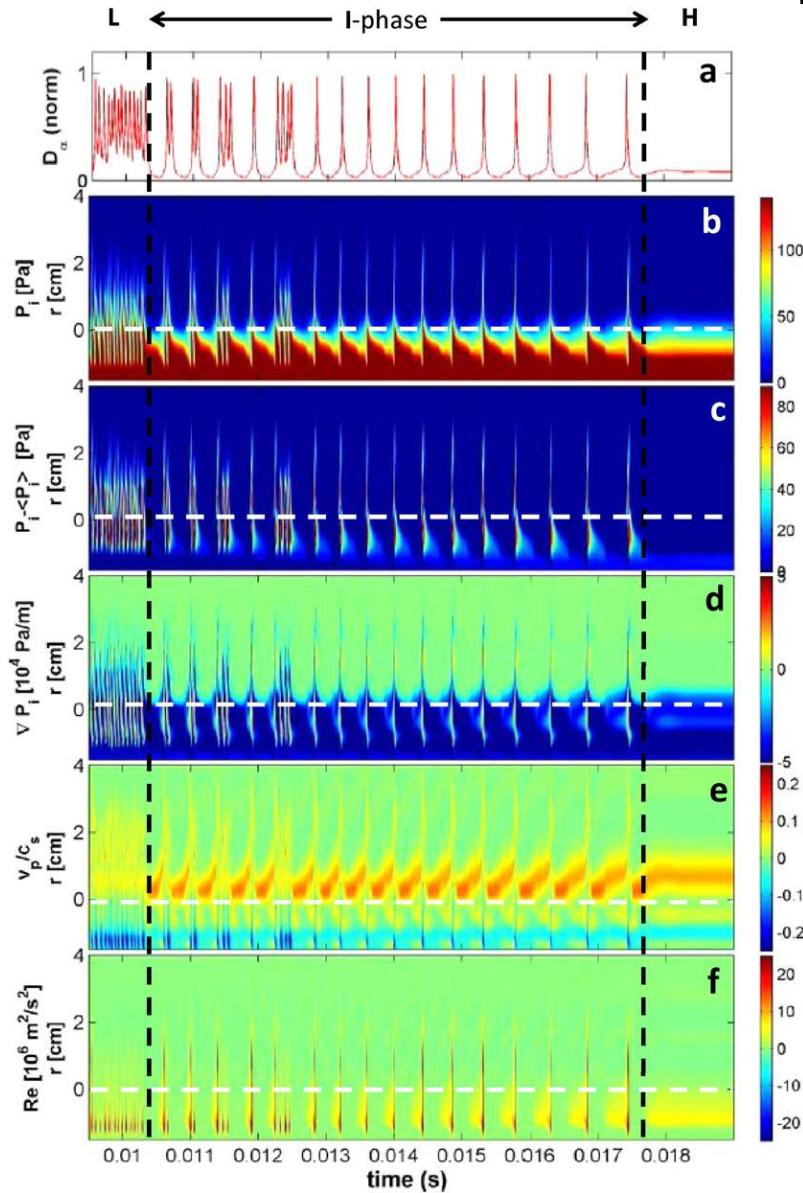
Xu et al. PRL 107, 125001 (2011); Nucl. Fus. 54, 013007 (2014)

# L-H-L transition: profiles

Poloidal and time average



# L – I – H simulation



I-Phase limit cycle oscillations, LCO–flow - turbulence interplay



Particle flux out of the confinement

Ion pressure profile

Fluctuations in the ion pressure profile

Ion pressure gradient

Zonal flow profile

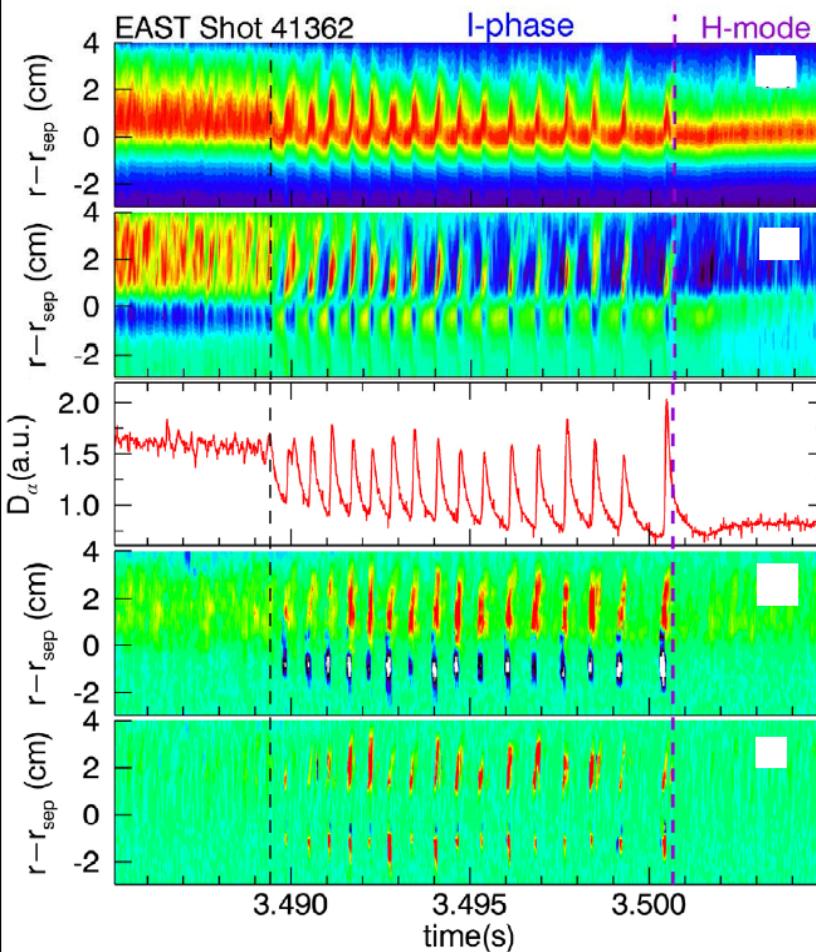
Reynolds stress

Rasmussen et al PPCF (2016) **58**, 014031

# L-I-H at EAST and in HESEL



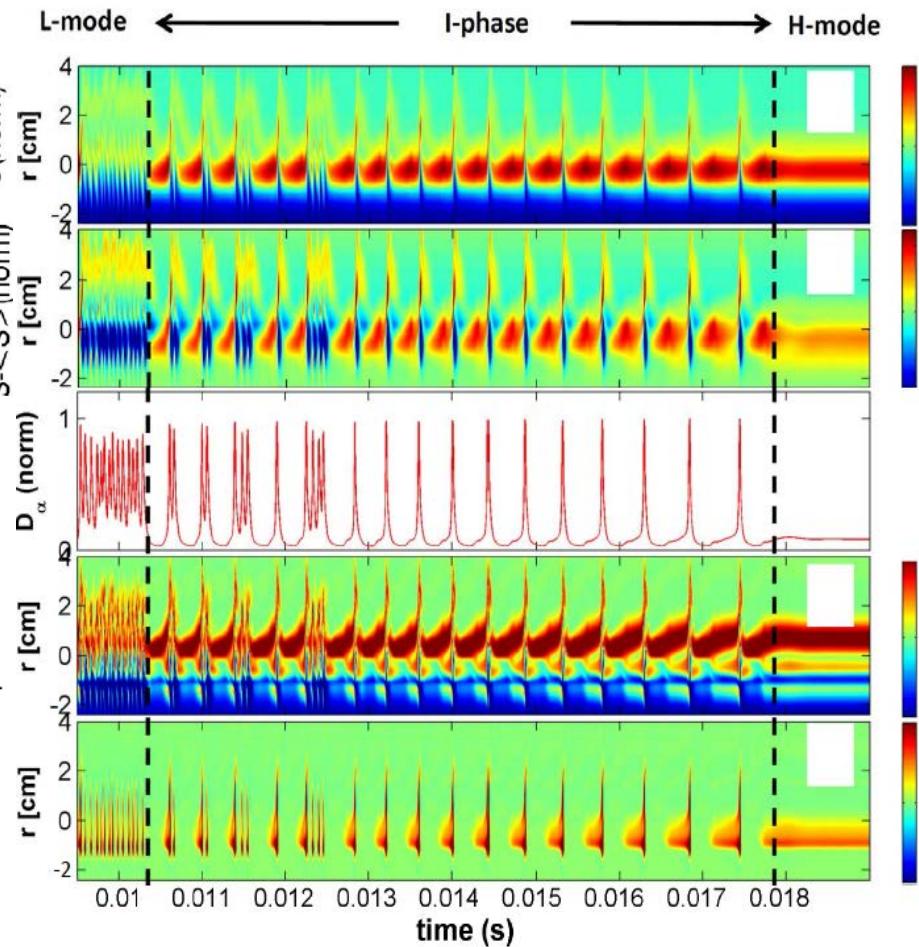
EAST



Gas puff imaging GPI – HeI line

Xu et al. NF 2014; Nielsen et al PLA (2015) 47-48, 3097.

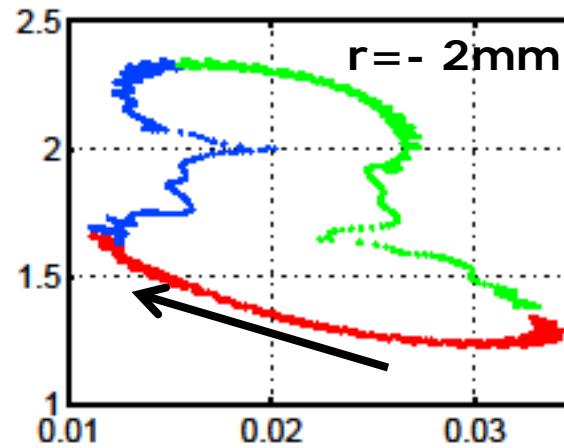
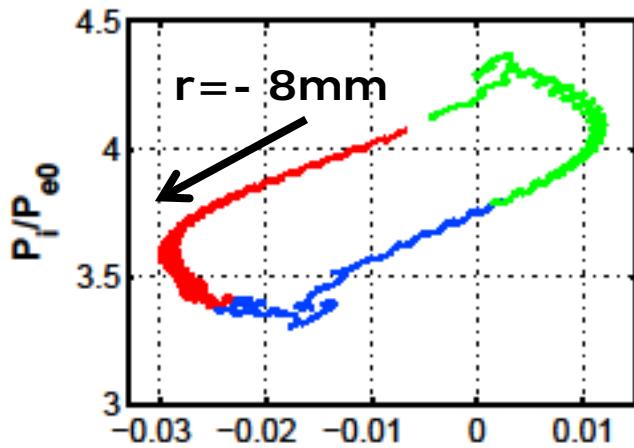
HESEL



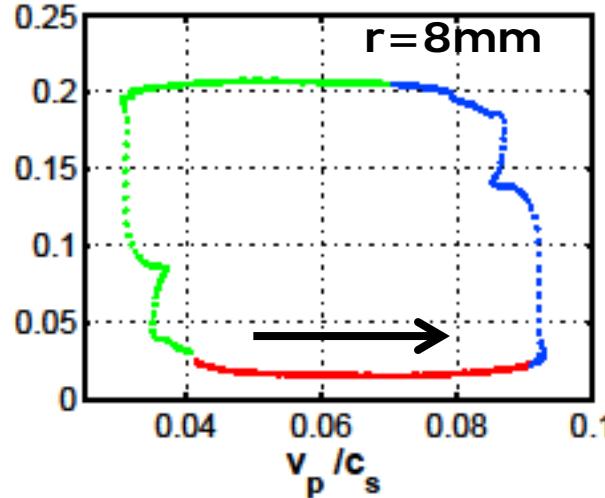
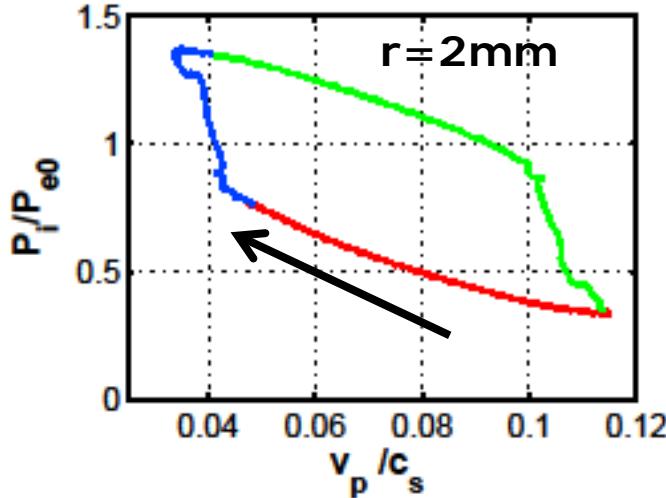
Synthetic GPI in HESEL provides S

S is emmision intensity, HeI line

# Lissajous diagrams – Causality of events



Ion pressure versus poloidal flow velocity for one LCO cycle



Three phases are observed, but the causality (rotation direction) depends on position

- Quiescent
- Turbulent burst
- Relaxing

Radial propagation of turbulence : *Estrada et al. Nucl. Fusion 55, 063005 (2015)*

# Summary

- HESEL model includes profile evolution, Edge-SOL coupling, ion energy dynamics, realistic collision parameters consistently experiments – no free parameters
- HESEL reproduces essential features of a L-H transition:
  - Robust transition – abrupt transition or slow transition with a dithering I-phase (LCO)
  - Power threshold density scaling
  - Comparisons with EAST experiments – qualitative and quantitative agreement
  - Dithering/LCO phase: reproduce evolution, dithering period (depends on ramp-up time), time scale, power threshold
- The model is simplified – drift fluid model, 2D and lacks real geometrical effects, electromagnetic effects.....

**Essential step connecting first principles models with heuristic 0D and 1D models to, allowing gauging parameters in 1D and 0D**

# Thank you for your attention

# EXTRA

# L-H transition -- Status



## Modelling:

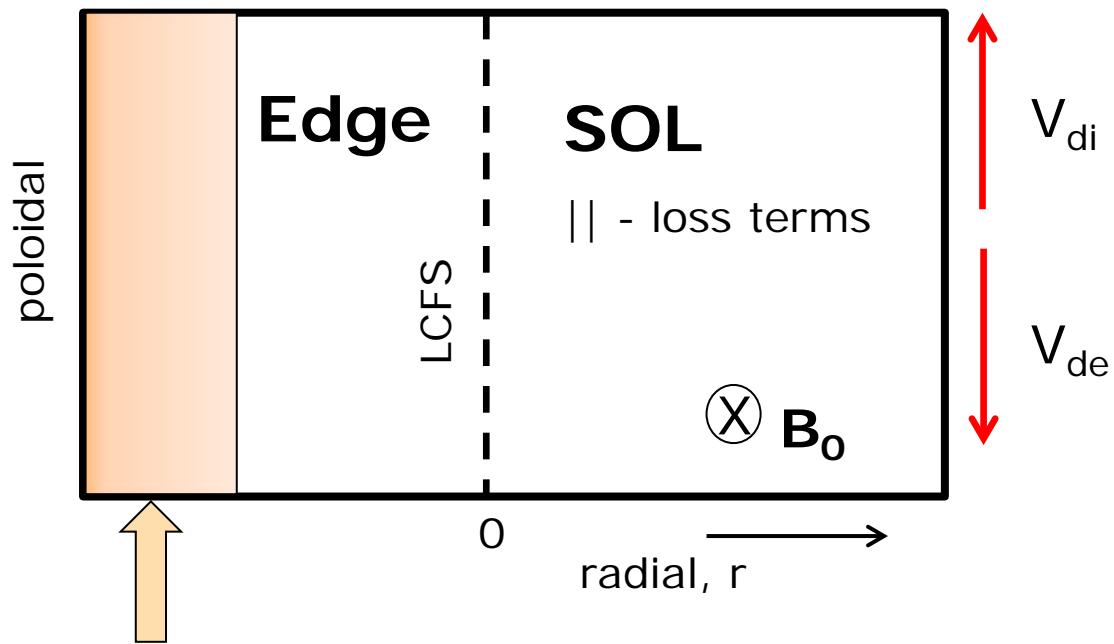
- **Based on “predator-prey” paradigm** á la Kim & Diamond PRL 2003, PoP 2003
- **0D**: *Malkov and Diamond PoP 2009, Dam et al. PoP 2013 ...*
- **1D**: *Miki et al. PoP 2012, Wu et al. NF 2015, Malkov et al. PoP 2015...* – heuristic models, qualitative descriptions
- **Fluid simulations**: *Drake et al. PRL 1998; Xu et al. PoP 2000; Thyagaraya et al. PoP 2010; Chone et al. PoP 2014; ...* demonstrate L-H transition in particular the formation of the edge transport barrier – no detailed scaling and comparisons with experiments.
- **Gyrokinetic simulations**: *Chang et al PRL 2017* demonstrate H-L transition mediated by zonal flows.

## Experiments:

- Recently experimental progress – advanced diagnostics: e.g., *Xu et al. PRL 2011, NF 2014; Schmitz et al PRL 2012; Cheng et al. PRL 2013; Kobayashi et al. PRL 2013; Cziegler et al. PPCF 2014; Ryter et al. NF 2014; Estrada et al. NF 2015, Schmitz et al Nucl. Fusion 2017*

# Set-up and parameters

- Slab geometry at outboard mid-plane
- Flux driven - interchange turbulence



Prescribed profiles – driving the fluxes

Ramp up of  $T_i$  profile part to increase the fluxes

## Parameters:

Typical conditions  
EAST (#41362):

$$n_0 = 1.5 \cdot 10^{19} \text{ m}^{-3} @ \text{LCFS}$$

$$T_{e0} = 20 \text{ eV} @ \text{LCFS}$$

$$T_{i0} = 20 \text{ eV} @ \text{LCF}$$

$$B_0 = 2.0 \text{ T} ; q_{95} = 4.0$$

$$R = 2.0 \text{ m} ; a = 0.5 \text{ m}$$

$$\Delta_{\text{SOL}} = 2.4 \text{ cm}$$

Wide parameter regime

Neo-classical transport and parallel damping rate coefficients calculated from plasma parameters @LCFS

Updated consistently