VIII-th International Conference "SOLITONS, COLLAPSES AND TURBULENCE: Achievements, Developments and Perspectives" (SCT-17) in honor of Evgeny Kuznetsov's 70th birthday.
May, 21-25, 2017. Chernogolovka, Russia

Topological 2D- and 3D-Dissipative Optical Solitons: Internal Structure, Symmetry, and Motion

Sergey V. Fedorov[1,2], Nikolay N. Rosanov[1-3,*], Nikolay A. Veretenov[1,2]

[1]Vavilov State Optical Institute,

[2]ITMO University,

[3]Ioffe Physical-Technical Institute.

Saint-Petersburg, Russia

[*]E-mail: nnrosanov@mail.ru

Light propagation in laser medium with saturable amplification and absorption

(3+1) Ginzburg-Landau-type equation for the electric field envelope *E*

diffraction

dichroism (angular selectivity)

dispersion of refractive index

dispersion of gain-losses

Diffusion coefficients $0 < d_{\perp,\parallel} \ll 1$

$$\frac{\partial E}{\partial z} = \left[(i + d_\perp)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + (i + d_\parallel) \frac{\partial^2}{\partial \tau^2} \right] E + f_{nl}(|E|^2)E$$

balance of gain and losses

$$f_{nl}(|E|^2) = \frac{g_0}{1 + |E|^2/\beta} - 1 - \frac{a_0}{1 + |E|^2}, \quad \tau = t - z/v_{gr}.$$

saturable gain

nonresonant absorption

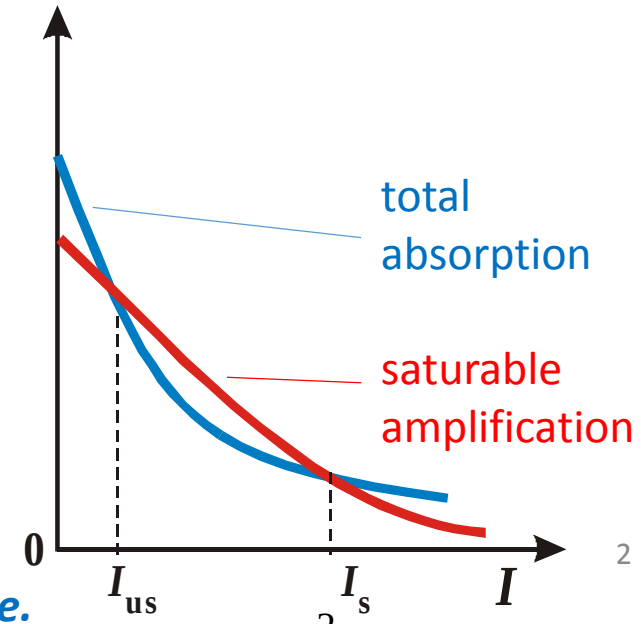
saturable absorption

group velocity

Homogeneous (translation invariance) weakly anisotropic medium or a long laser with saturable absorption. One-component system (linear polarization).

$f_{nl}(0) < 0$ - trivial solution $E = 0$ (non-lasing mode) is stable

$f_{nl}(I_{us}) = f_{nl}(I_s) = 0$ - hard excitation of lasing, bistability



$I = |E|^2$ - intensity

Laser (dissipative) soliton = an island of lasing on the background of stable non-lasing mode.

[Rosanov, Fedorov (1992)]

Early: 1D-, 2D-, and simplest 3D-solitons.

Now: 3D-Topological Solitons using 2D-solitons

- L. D. Faddeev, “Quantization of Solitons“, Princeton preprint IAS-75-QS70, Institute for Advanced Study, Princeton (1975). *– multicomponent media*
- Y. V. Kartashov and B. A. Malomed and Y. Shnir and L. Torner, “Twisted Toroidal Vortex Solitons in Inhomogeneous Media with Repulsive Nonlinearity”, Phys. Rev. Lett. **113**, 264101 (2014). *– inhomogeneous media*
- N. N. Rosanov, N. V. Vysotina, A. N. Shatsev, A. S. Desyatnikov, and Y. S. Kivshar, "Knotted Solitons in Nonlinear Magnetic Metamaterials", Phys. Rev. Lett., **108**, 133902 (2012). *– periodic inhomogeneity, discrete knotted solitons*
- D. Mihalache, D. Mazilu, F. Lederer, et al. “Stable Vortex Tori in the Three-Dimensional Cubic-Quintic Ginzburg-Landau Equation”. Phys. Rev. Lett. **97**, 073904 (2006). *– extremely large dissipation*
- D. Mihalache, "Multidimensional Localized Structures in Optical and Matter-Wave Media: A Topical Survey of Recent Literature", Romanian Reports in Physics, **69**, 403 (2017). *– nice review*

Transforming 2D- to 3D-topological soliton structure: Preparation of the initial field distribution (at $z = 0$)

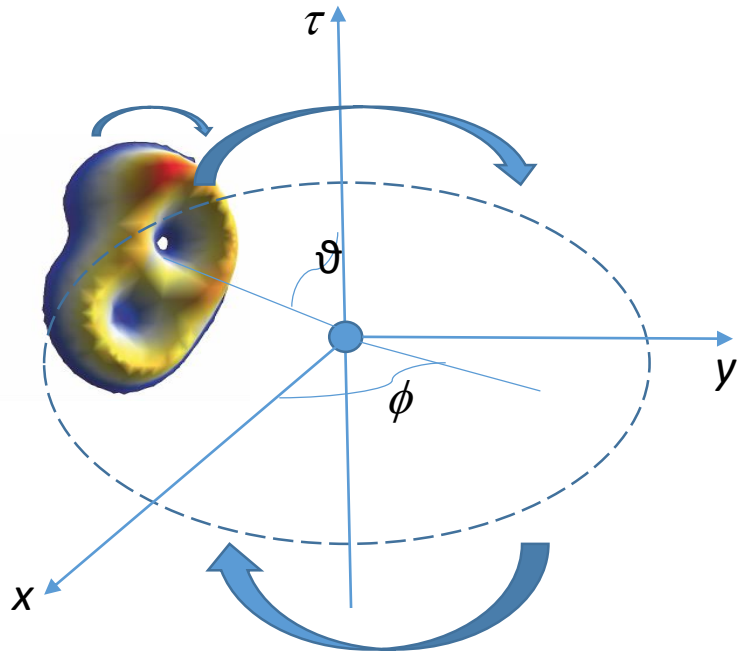
There are known various 2D-laser structures: single fundamental and vortex solitons and their complexes with weak and strong coupling.

1st step: we use one of them in the plane (x,τ) of 3D-space.

$$d_{\parallel} = d_{\perp} = d$$

2nd step: we rotate this structure (ϕ) around the axis τ.

The structure's center trajectory is a circumference.



3rd step: simultaneously, we rotate (twist) this structure in the azimuthal direction (ϑ) in such a way to get a closed figure in result of these two rotations.

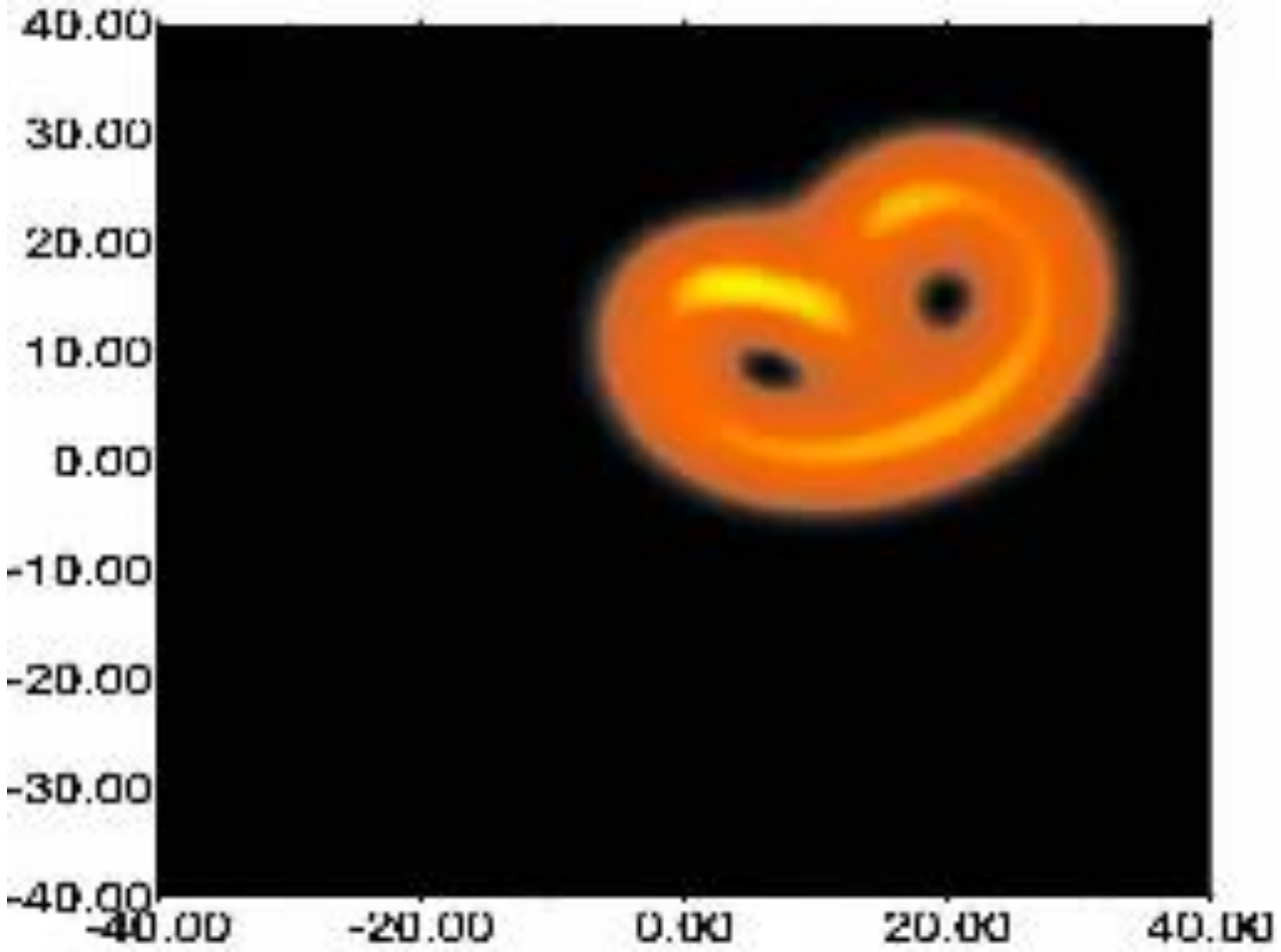
4th step: we include a vortex with the help of multiplayer $\exp(im\phi)$.

5th step: transient (evolution with z).

Metastable or stable structure?

Analogy: the talk of Prof. A.C. Newell here.

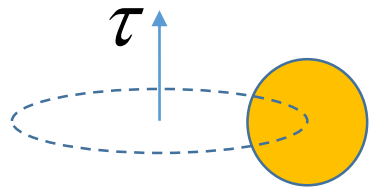
Example of 2D-vortex structure: Strongly coupled two vortex solitons



Moon-like motion: Center revolves along a circle, and the structure rotates (twists) with the same period (one of four scenarios of the "Eulerian mechanics" for solid-like soliton complexes, depending on complex' symmetry).

Search for topological 3D-dissipative solitons

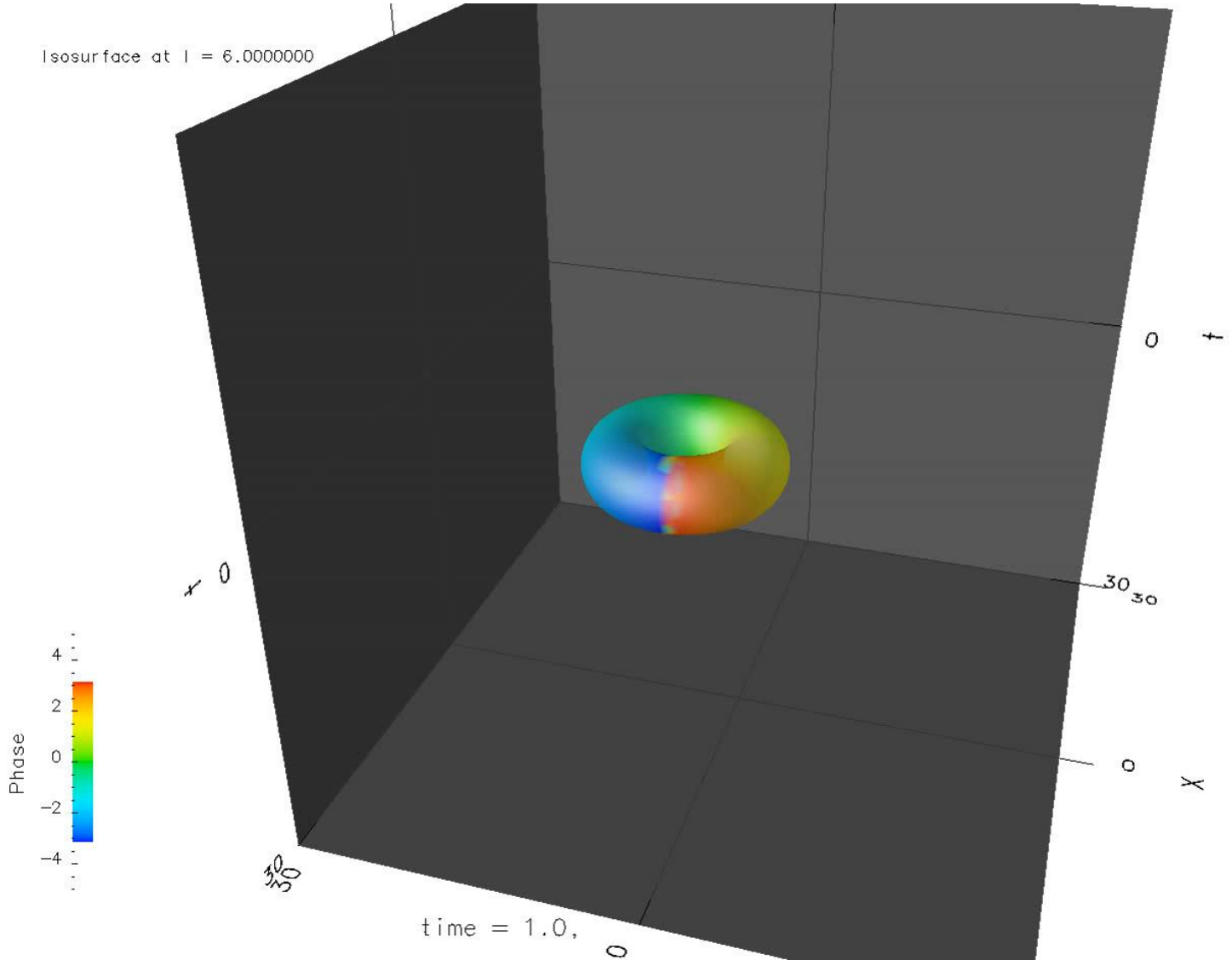
$$d_{\square} = d_{\perp} = d$$



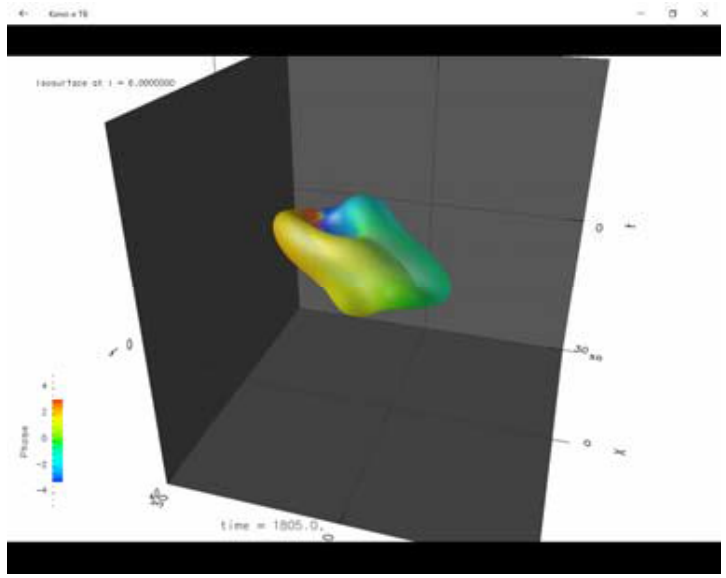
Initial 2D-distribution:
a fundamental soliton;

$$m_{2D} = 0, \quad m_{\tau} = 1$$

Toroidal structure with a vortex line?



Final structure

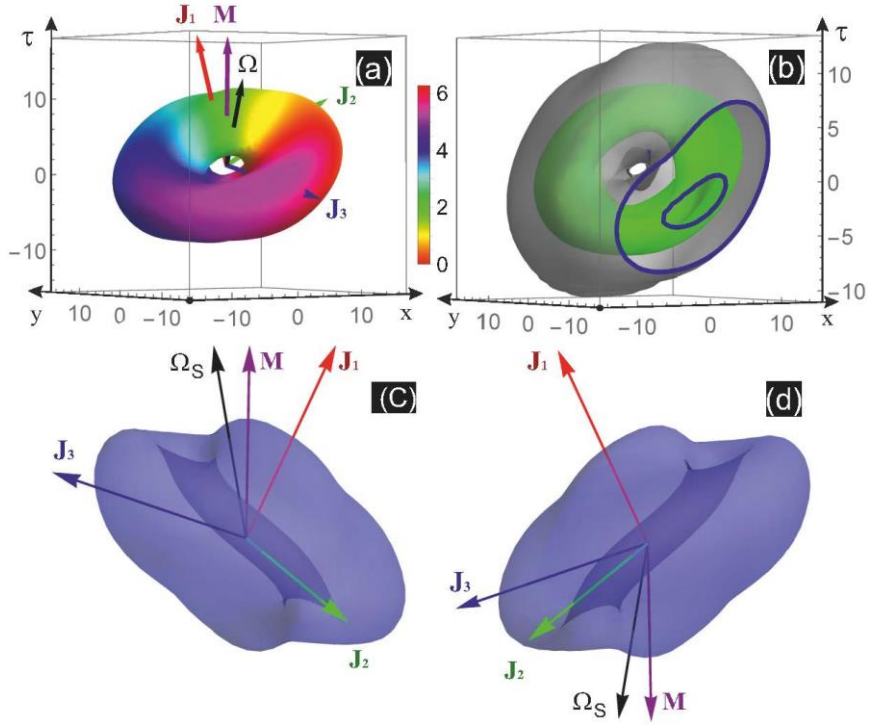


Но пораженья от победы
Ты сам не должен отличать.
Б. Пастернак

You should not distinguish yourself
Between a defeat and a victory.
B. Pasternak

“Solid” asymmetric rotating 3D-dissipative soliton

$$d_{\parallel} = d_{\perp} = d$$

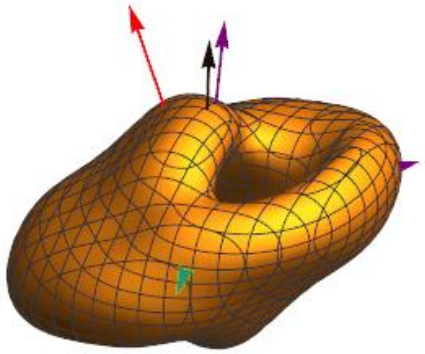


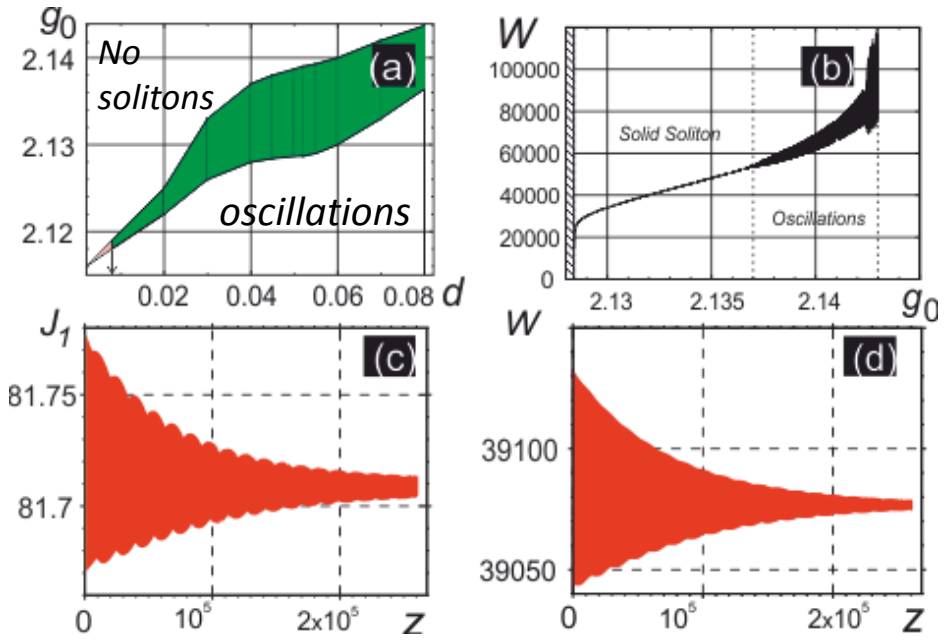
(a) “Instantaneous” toroidal isointensity surface. The angular velocity of rotation is constant. **J**1,2,3 are the principal axes of inertia tensor with corresponding moments J1,2,3, **M** is torque. (b) Two toroidal surfaces, one embedded in the other, of vanishing divergence of energy flux, div **Sr** = 0, separating domains of energy sources and sinks. (c) One-half of the soliton in (a). (d) Mirror image of (c).

$$\mathbf{M} = \int \mathbf{r} \times \mathbf{S}_{\mathbf{r}} d\mathbf{r}, \mathbf{r} = (x, y, \tau),$$

$$\mathbf{S}_{\mathbf{r}} = \mathrm{Im}(E^* \nabla_{\mathbf{r}} E),$$

$$J_{ij} = \int (\delta_{ij} r_j^2 - r_i r_j) \, | E(\mathbf{r}, z) |^2 \, d\mathbf{r}$$

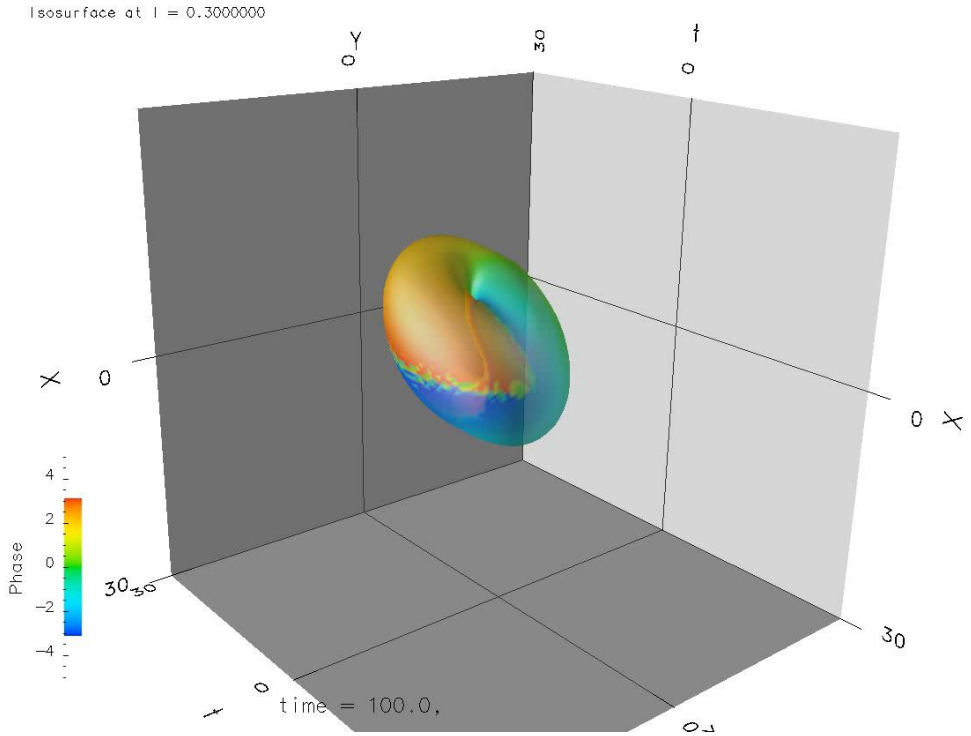




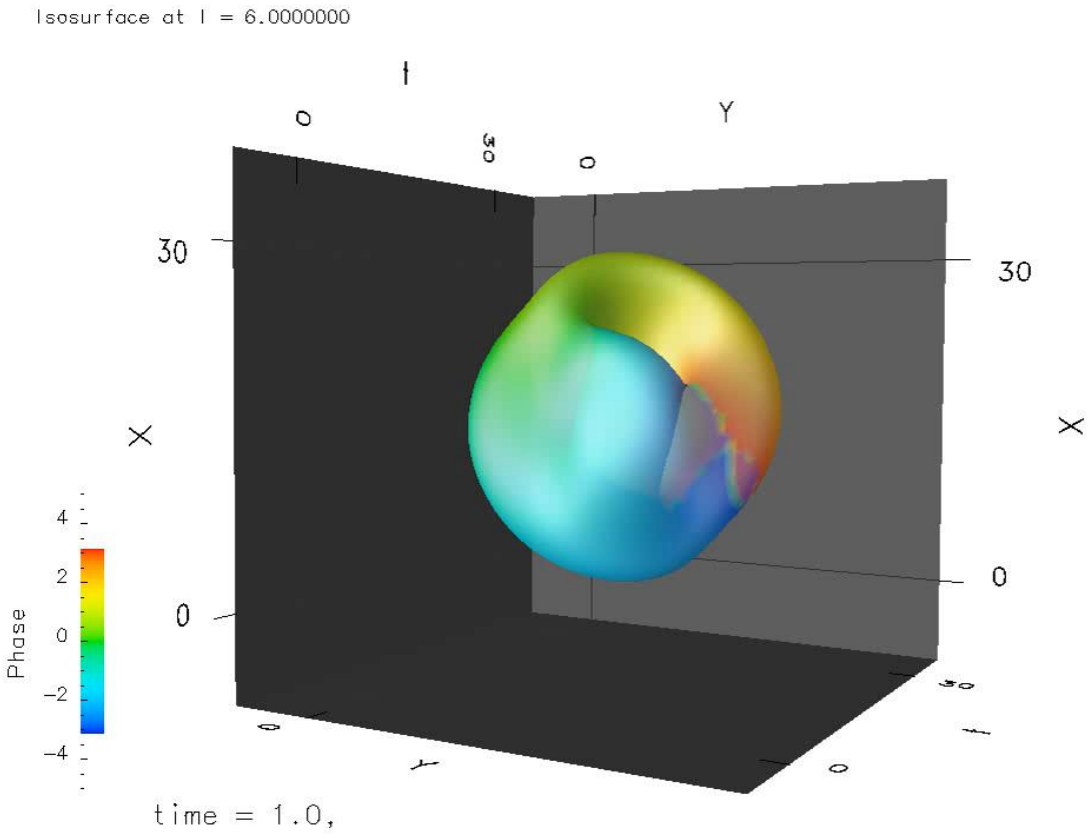
(a) Stability domain of “solid” solitons. (b) Soliton energy *W* vs. gain g_0. (c),(d) – transient for the maximum principal momentum of inertia J_1 (c) and for energy *W* (d)

Scenarios of destabilization for decrease and increase of diffusion or gain

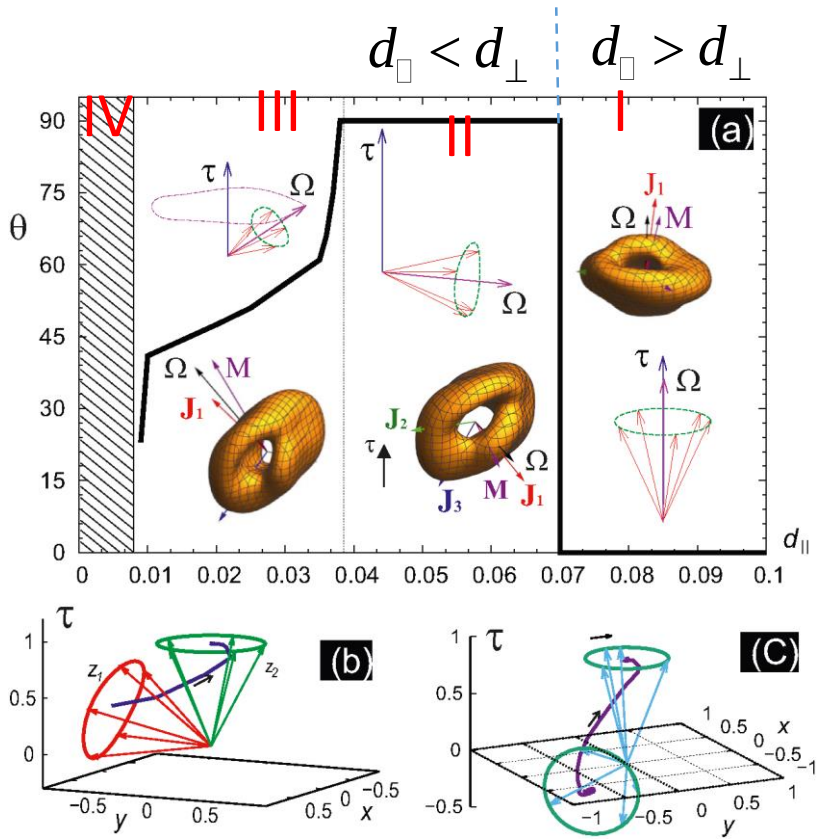
decrease



increase



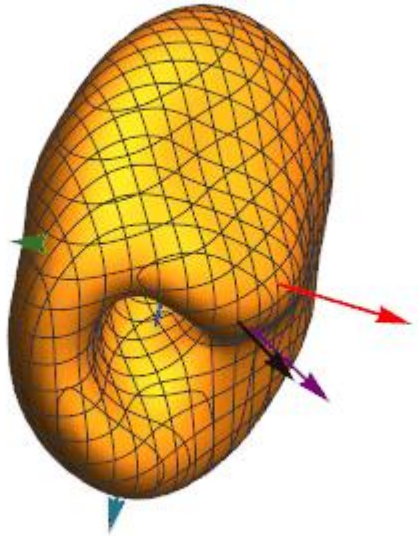
Asymmetric rotating and precessing solitons (precessons)



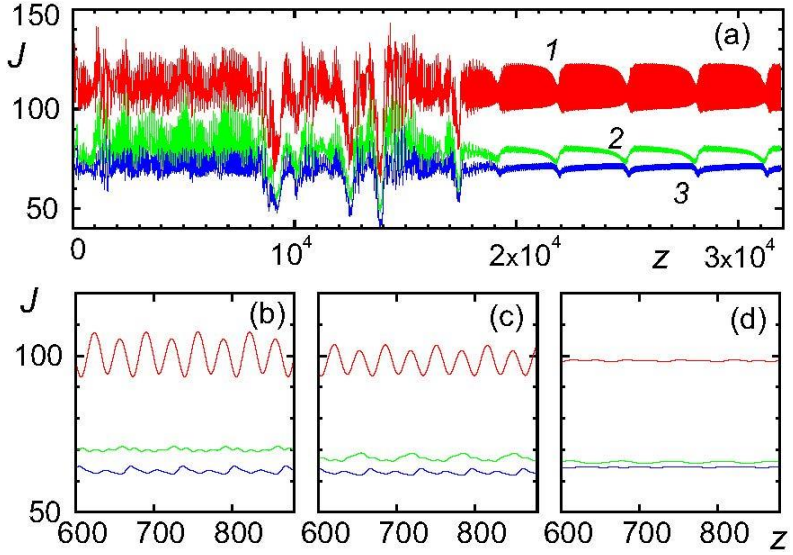
(b) Transient of Ω in domain (a), I.
(c) Transient of Ω in domain (a), II

θ - angle between axis τ and the principal axis of inertia $\mathbf{J}_1$ (with largest moment)

(a) Dependence of θ on the diffusion coefficient $d_\parallel$ for fixed $d_\perp$ and g_0.
I. "Solid" rotating soliton, angular velocity Ω is parallel to axis τ. $d_\parallel > d_\perp$
II. Rotating, precessing, and weakly oscillating soliton, Ω is orthogonal to τ.(*)
III. Similar to II with intermediate orientation of angular velocity Ω .
IV. No localized structures.

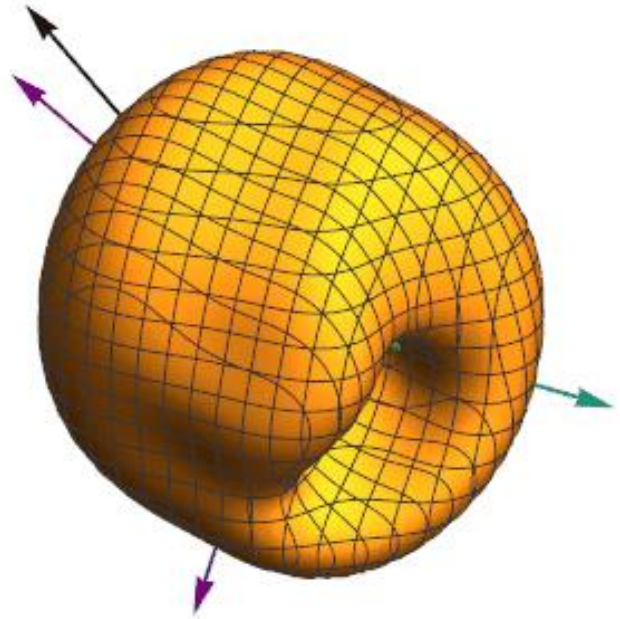


II $d_\parallel < d_\perp$

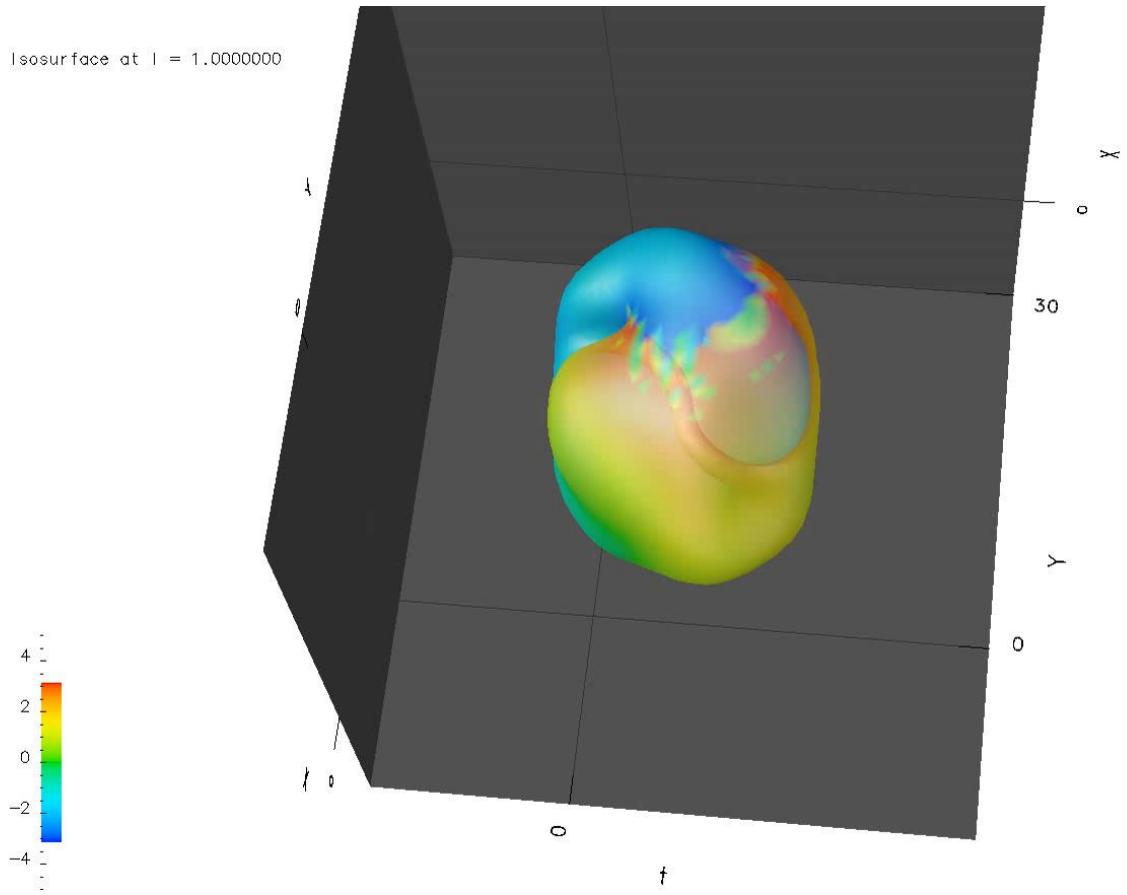


Dependence of the principal moments of inertia $J_{1,2,3}$ on propagation distance z for (a) chaotic-like localized state, (b) precessing soliton, (c0) periodically oscillating soliton, and (d) "solid" soliton

Weakly oscillating and slowly precessing soliton



Localised structure with chaotic dynamics

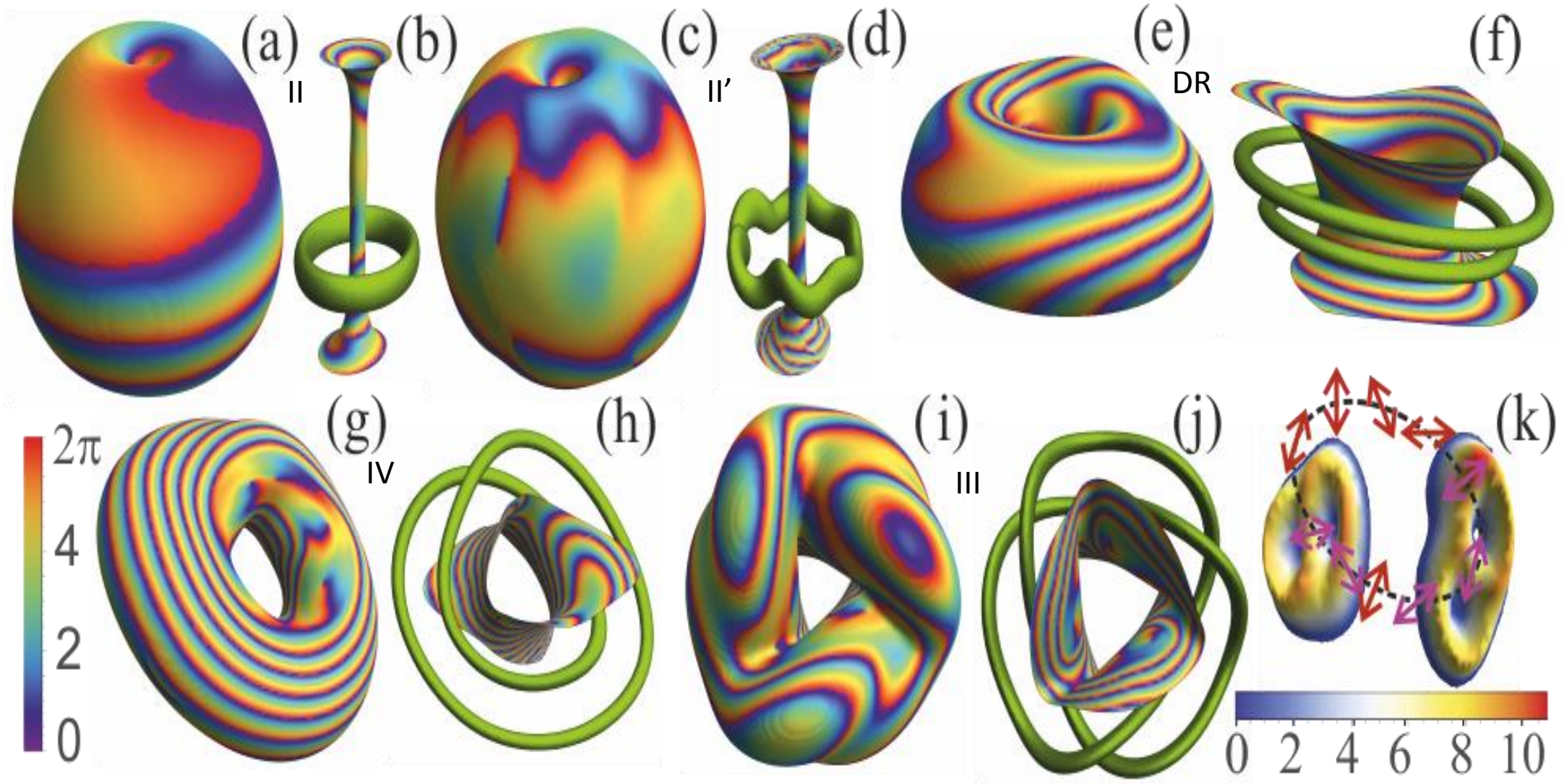


Single unclosed infinite vortex line

Using 2D-*vortex* structures to generate 3D-topologic structures

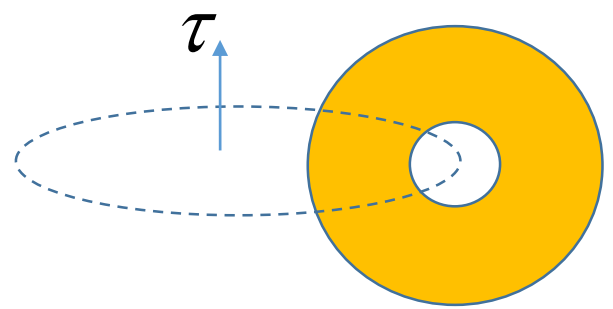
Traces of 2D-vortices form closed vortex loops.
Rotation and twisting braid the vortex lines

$$d_{\square} = d_{\perp} = d$$

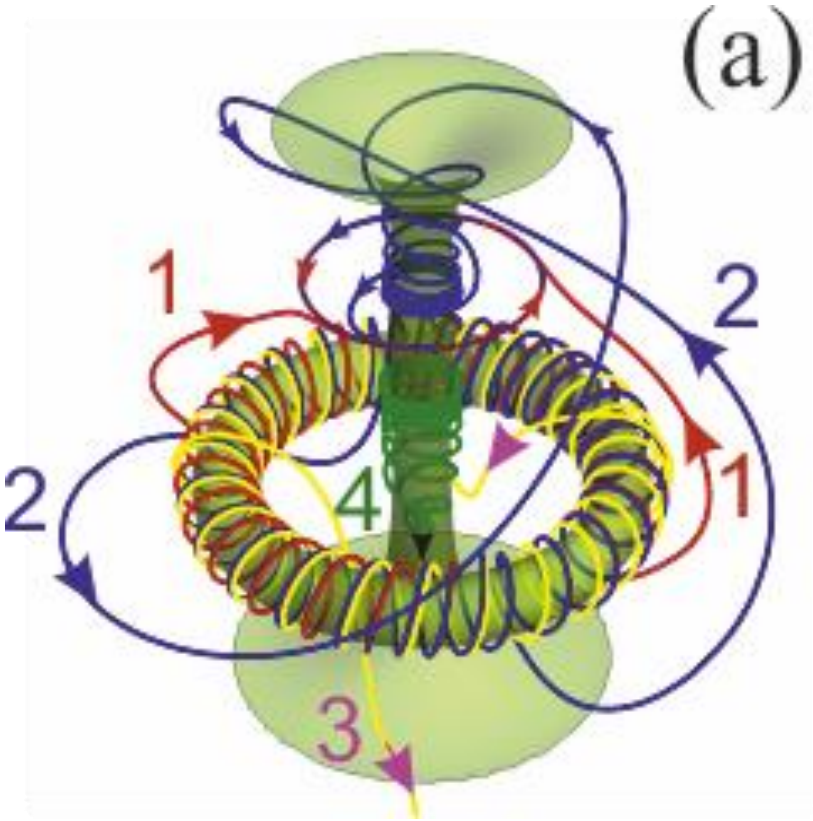


Twisted torus ("Apple"). 2D-generating structure: A vortex 2D-soliton

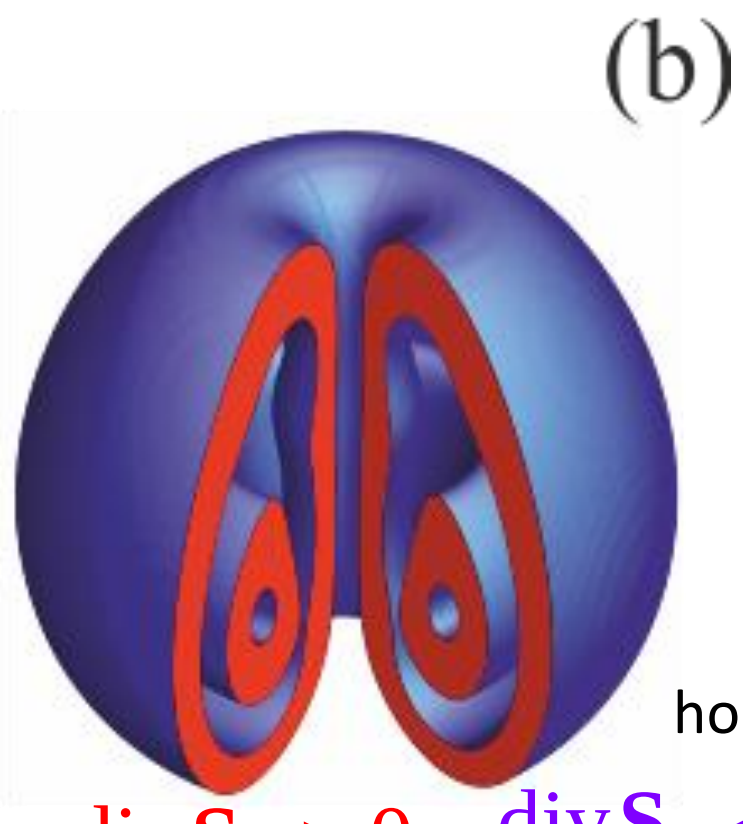
$m_{2D} = 1, \quad m_\tau = 1$



(a) Skeleton – two vortex lines, and two lines 1,2 of energy flux (Poynting vector) → *strong coupling of vortex lines*



Topological charges = 1

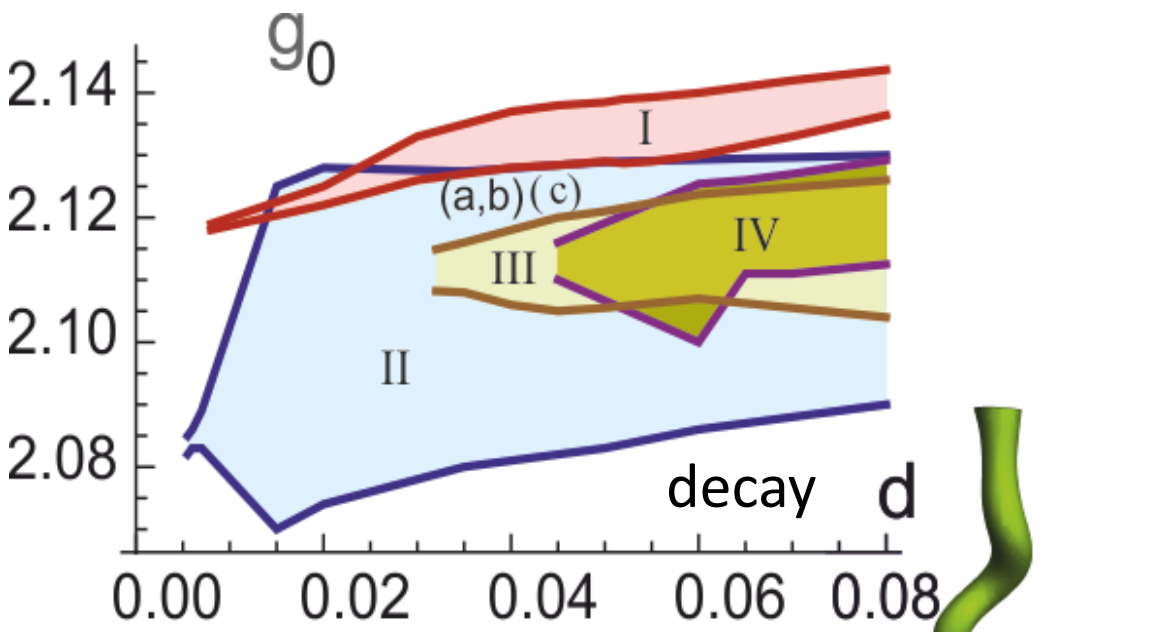


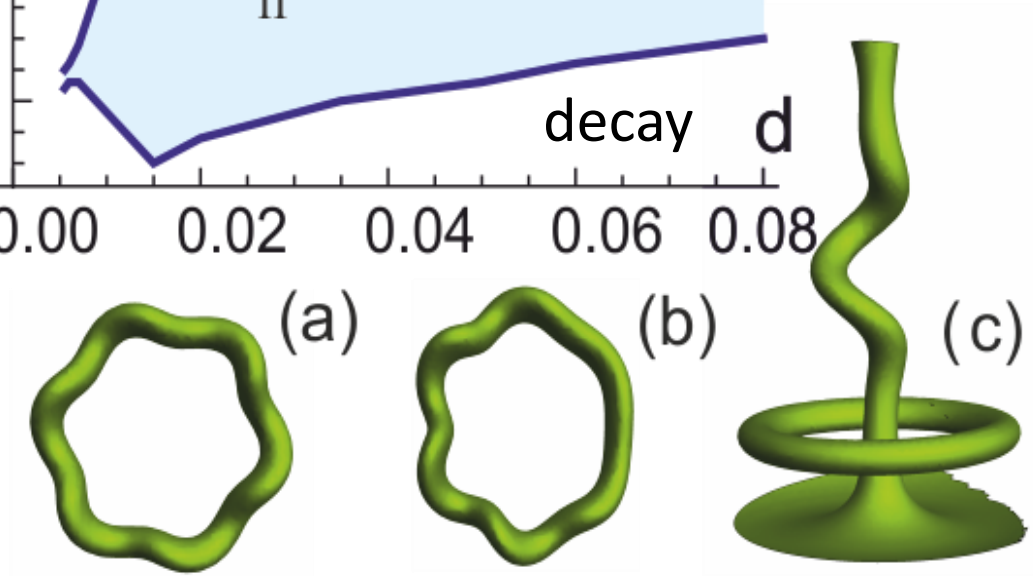
(b) Energy flux divergence

hollow tori

$\mathrm{div}\,\mathbf{S_r} > 0 \quad \mathrm{div}\,\mathbf{S_r} < 0$

Domains of stability of localized topological structures at the parameters' plane "diffusion d – gain g_0"





Waved closed vortex line

Waved unclosed (infinite) vortex line

I - **precessons**

II - twisted tori ("apples"), (a),(b),(c)

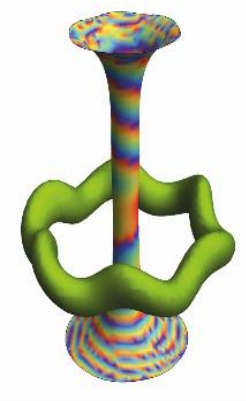
III - trefoil

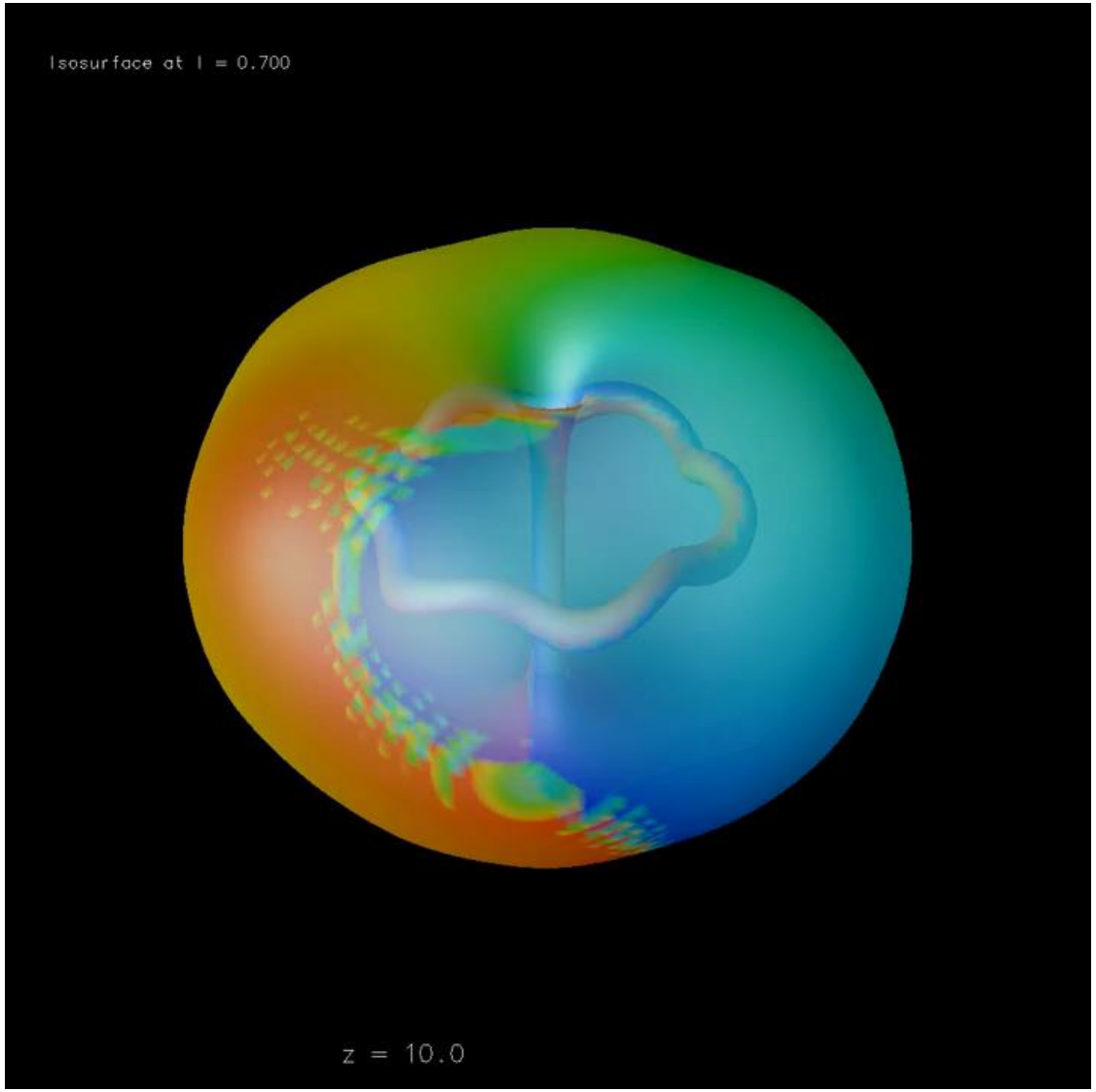
IV – unknot

Not shown **– double ring**

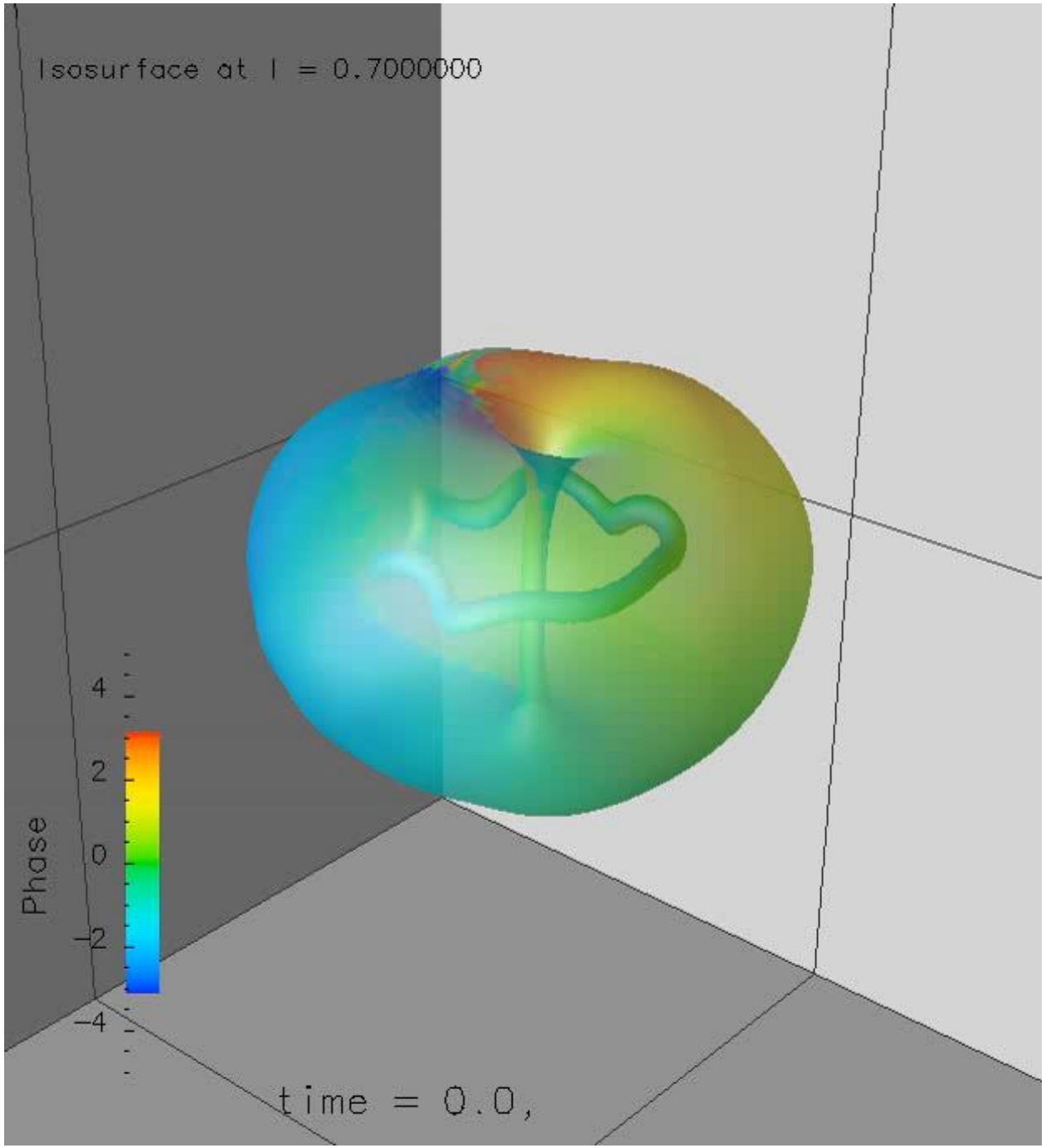
Overlapping of the domains

Apple – 6

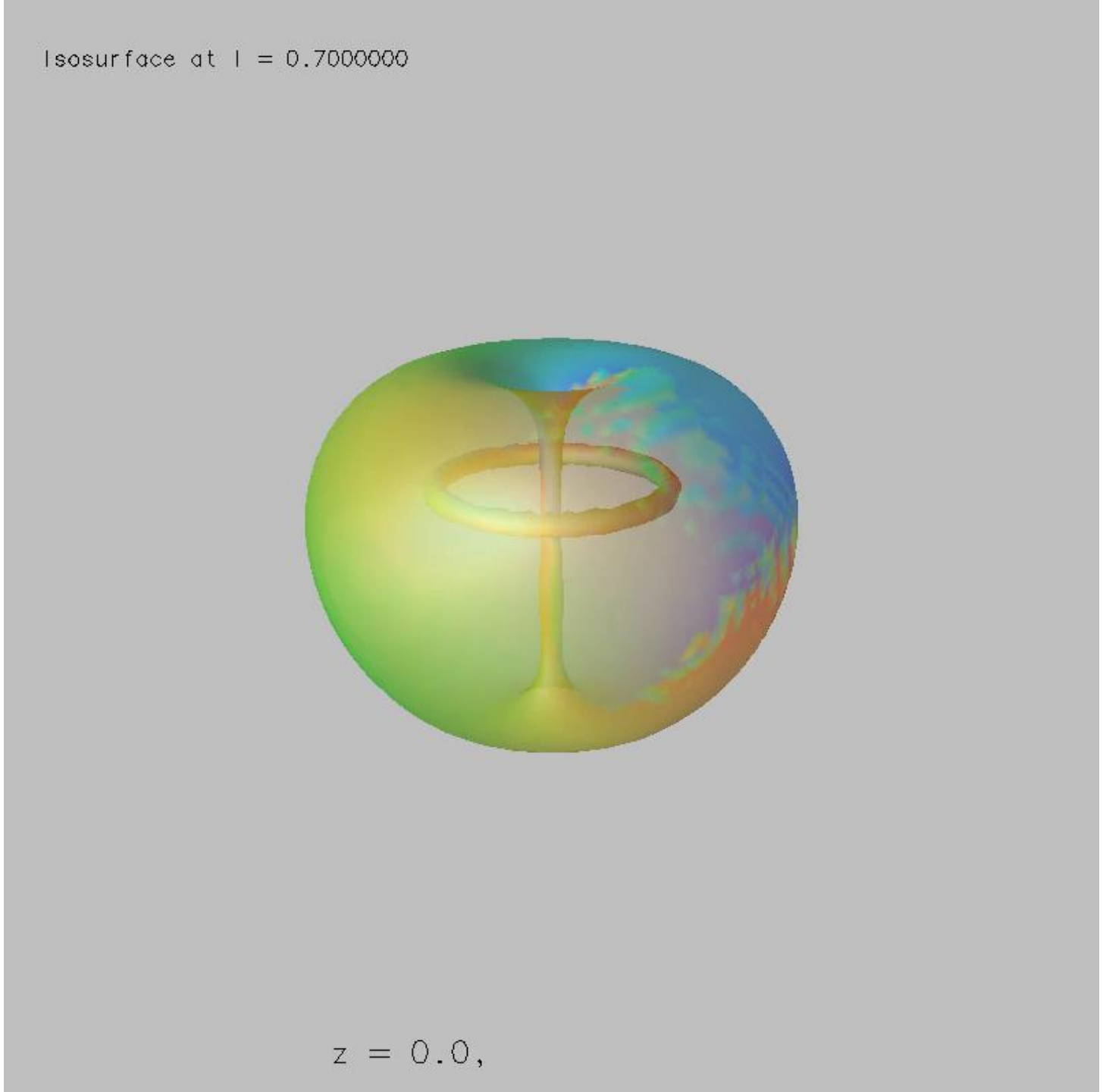




Apple 5-6



Apple decay due to the infinite vortex line waving



Effect of $d_{\parallel} \neq d_{\perp}$

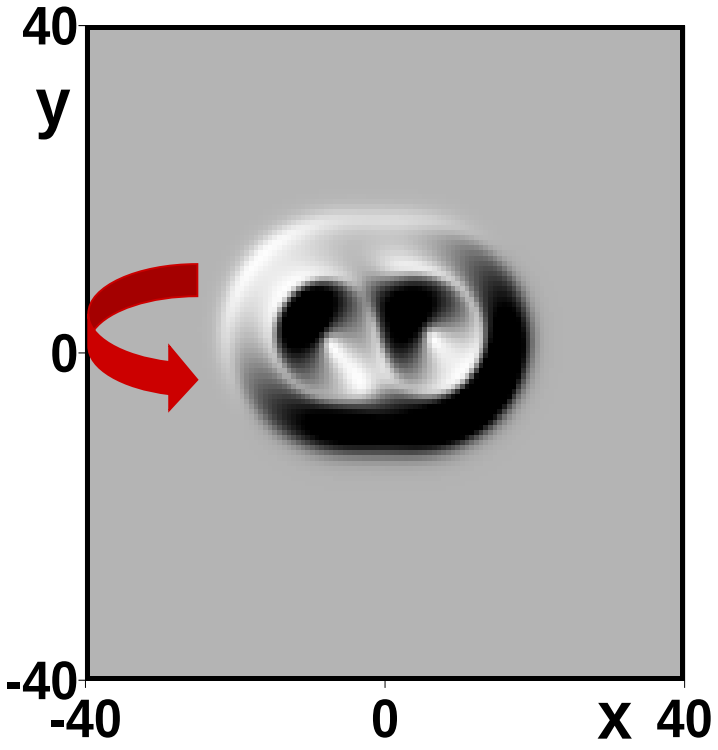
Similarly to the case of the “precessons”:

In the degenerate case, $d_{\parallel} = d_{\perp}$, orientation of angular velocity is arbitrary.

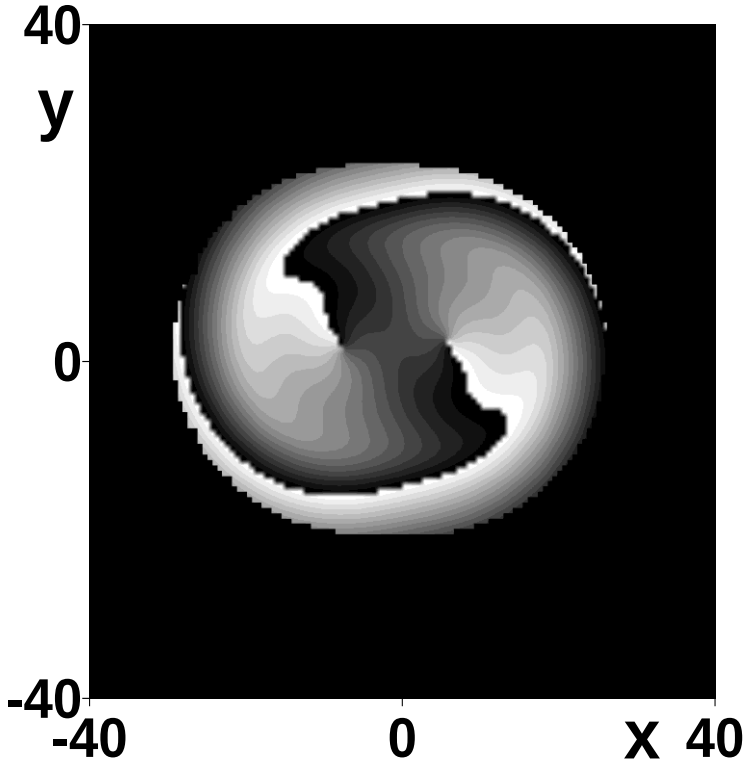
For $d_{\parallel} \neq d_{\perp}$, there is a transient with angular velocity reorientation along the direction with greater diffusion coefficient $d_{\perp}$ or $d_{\parallel}$.

Generating 2D-structure: A pair of two strongly coupled vortex solitons, $m_1 = m_2 = 1$

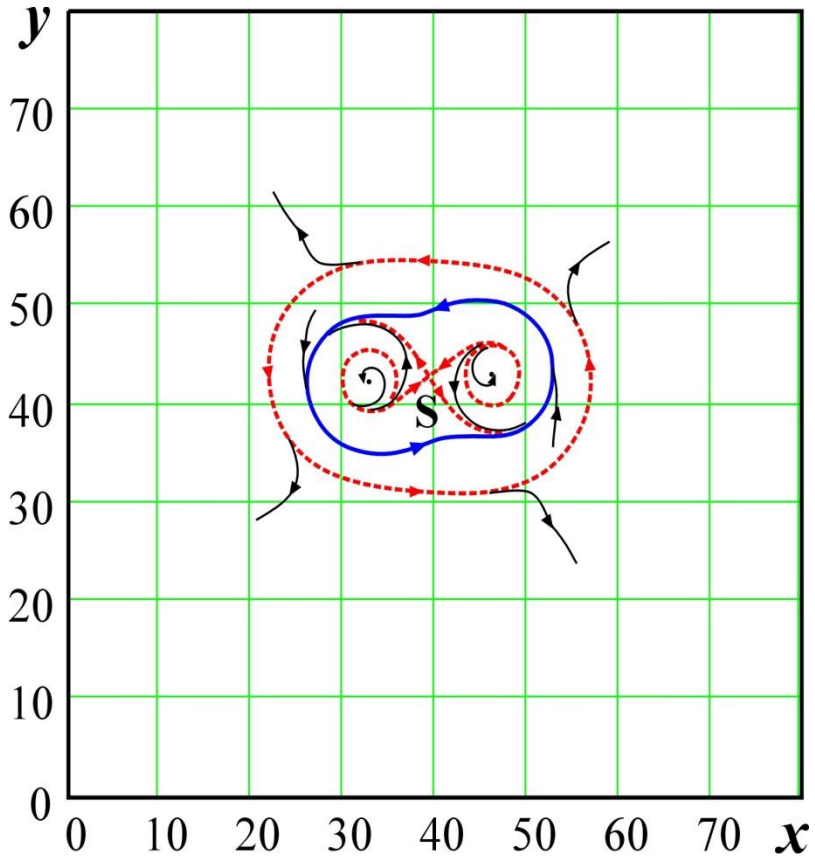
Intensity



Phase



Instantaneous energy fluxes



strong coupling of vortices

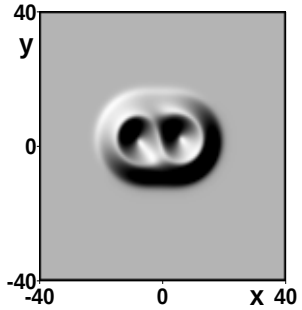
Central symmetry, rotation around the motionless symmetry center

Double Rings: Strong coupling of closed vortex lines

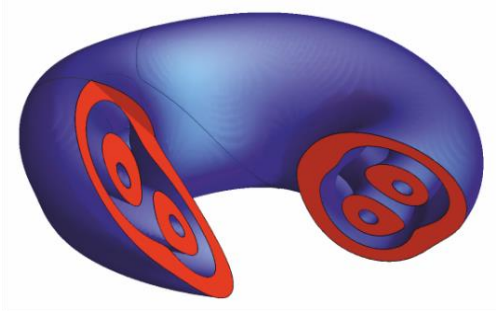
$m_{2D} = 1+1, \quad m_\tau = 3,$

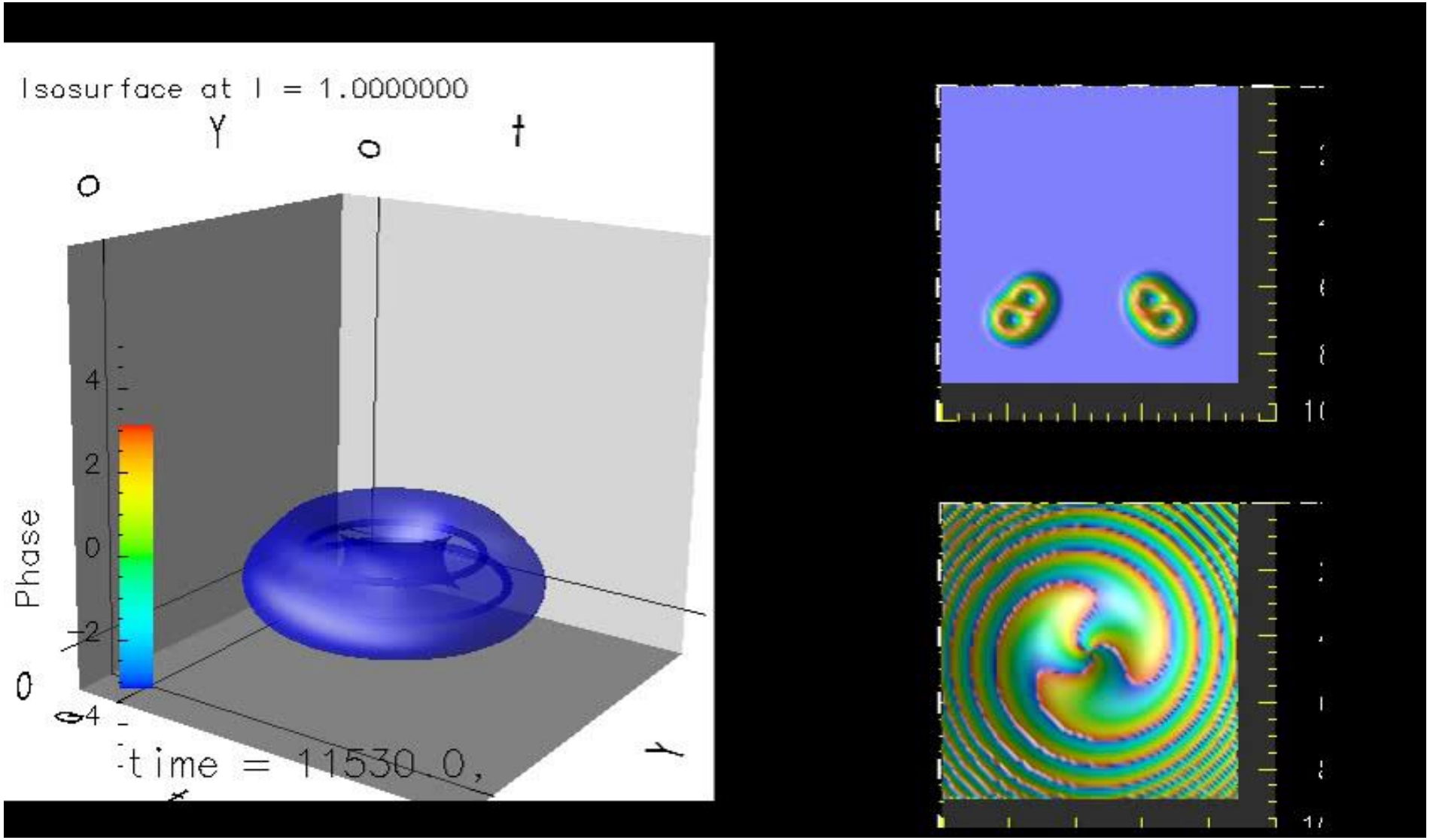
no twist

2D-generating structure: two strongly coupled vortex solitons with topological charges 1 +1



$\mathrm{div}\,\mathbf{S_r} > < 0$

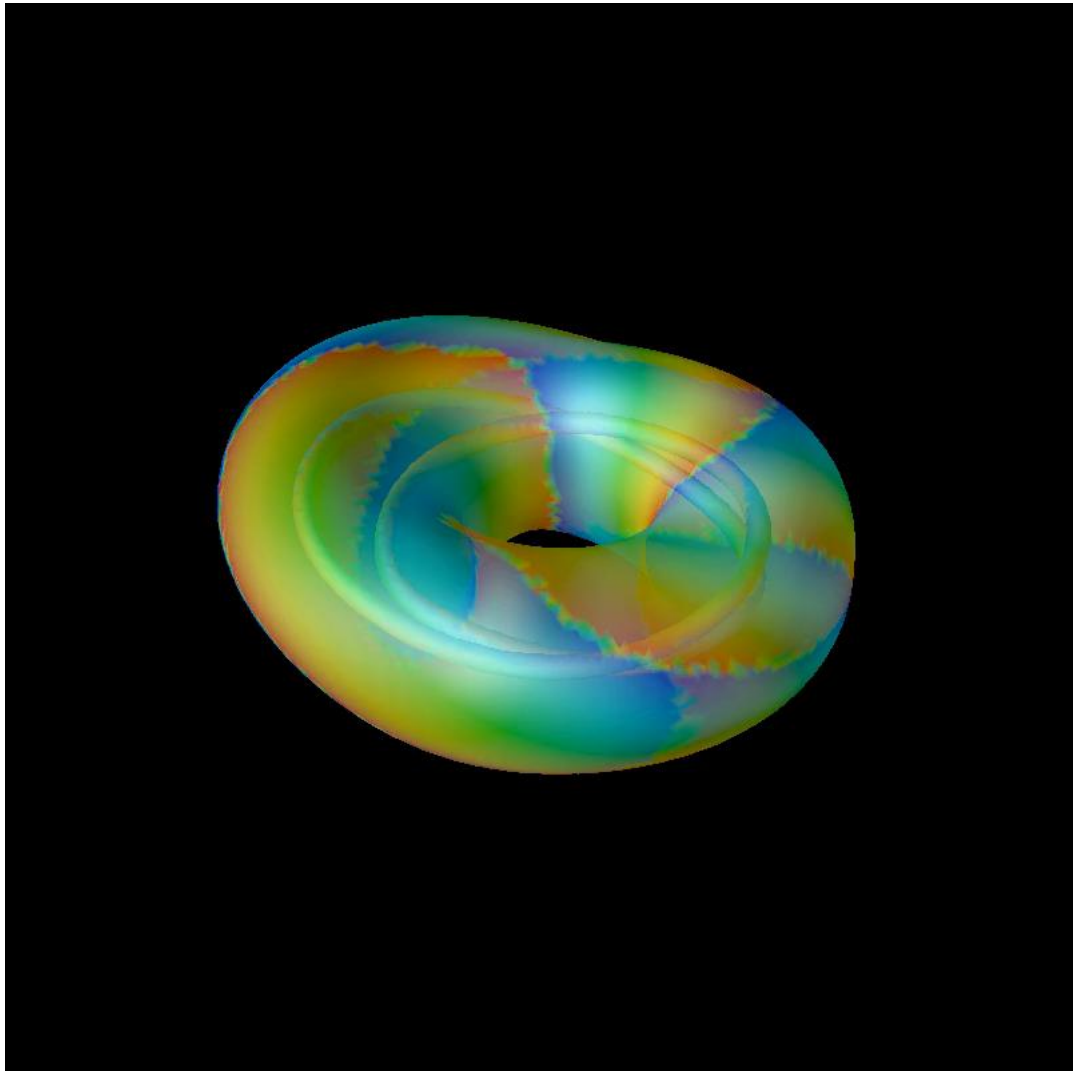


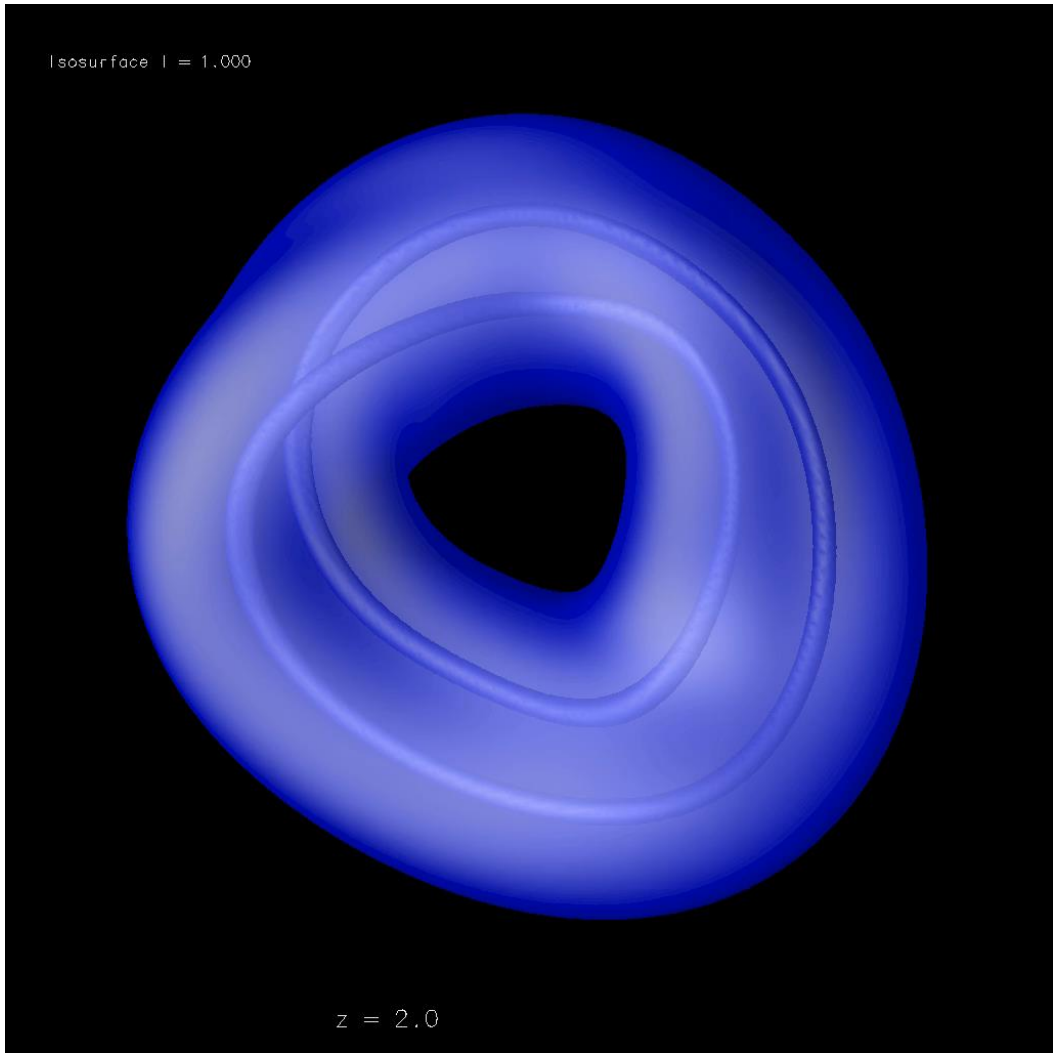


Unknot

$m_{2D} = 1+1, \quad m_\tau = 3,$

twist 1/2

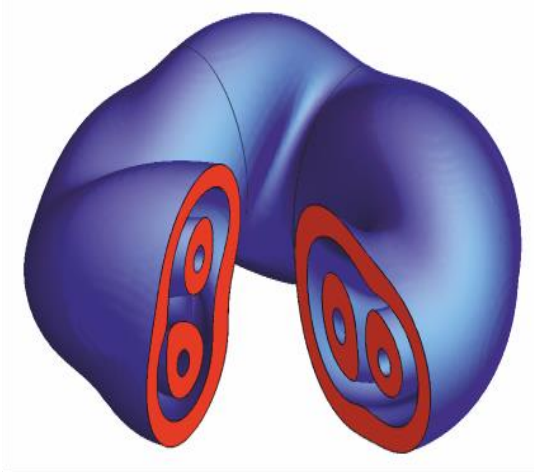




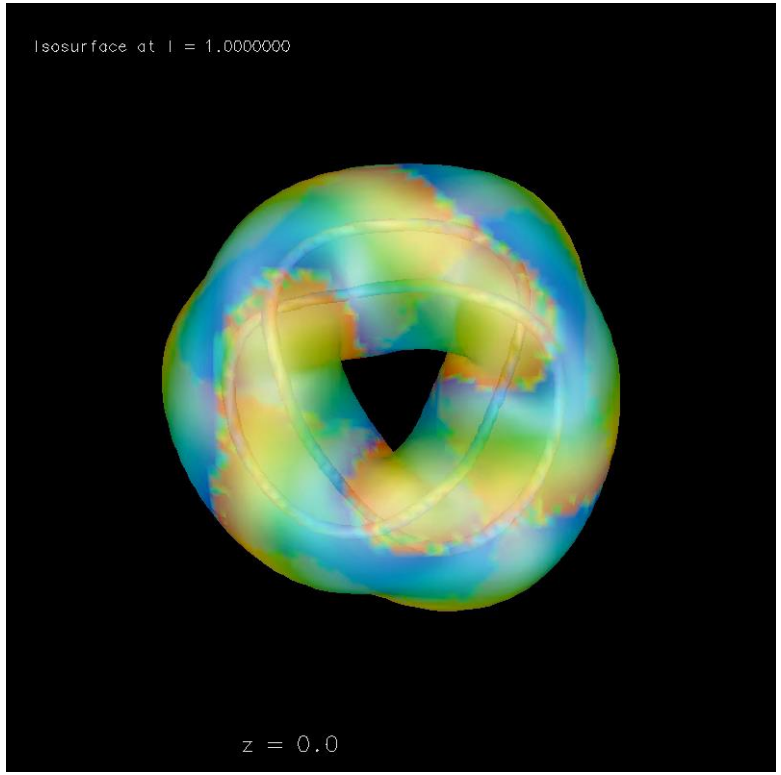
Trefoil

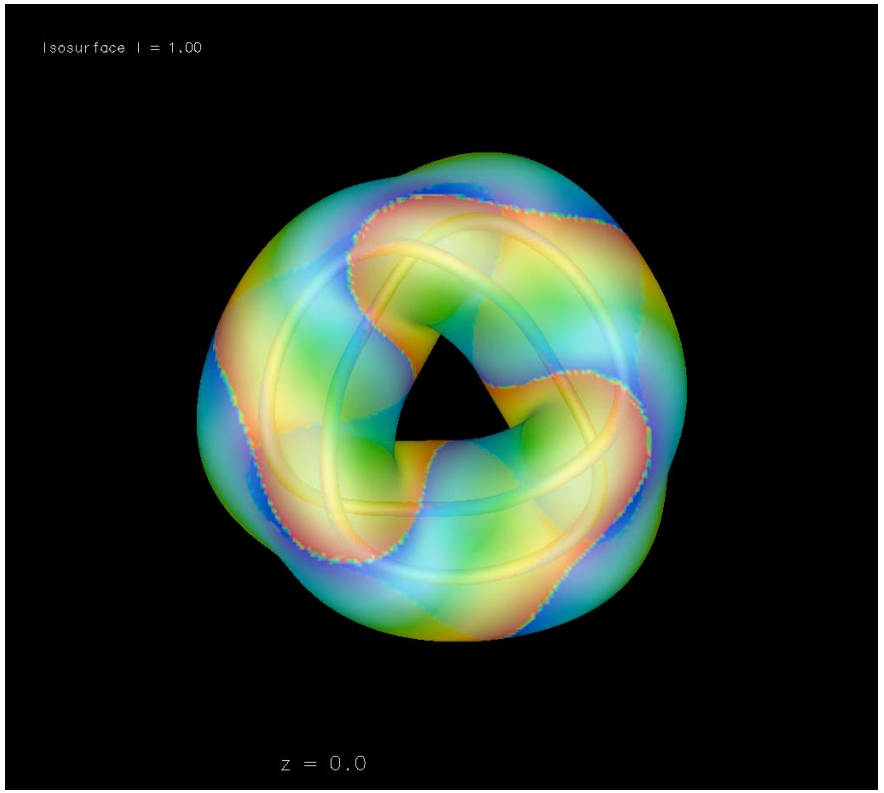
$m_{2D} = 1+1, \quad m_\tau = 3,$

twist 3/2



$\mathrm{div}\,\mathbf{S_r} > < 0$





Conclusion

2D-laser (dissipative) solitons and their complexes generate a new wide class of 3D-topological solitonic structures in homogeneous one-component nonlinear active media. The structures include an infinite unclosed and one or a number of unknotted and knotted vortex lines (loops). In optics, the necessary initial structures can be formed in laser media or in large-size lasers with saturable absorption with the help of spatial light modulators.

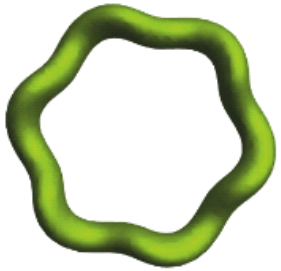
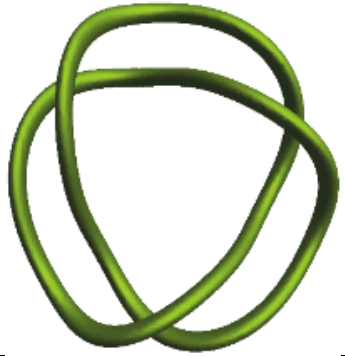
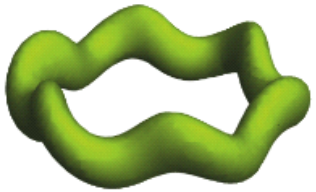
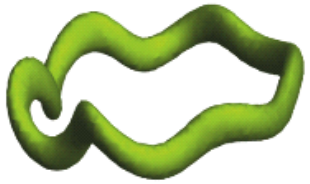
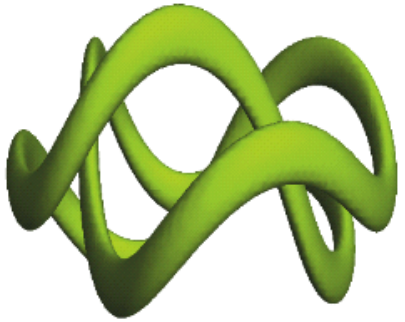
Some references

N. N. Rosanov, *Dissipative Optical Solitons. From Micro- to Nano- and Atto-* (Fizmatlit, Moscow, 2011, in Russian).

N. N. Rosanov and S. V. Fedorov, "Topology of energy fluxes in vortex dissipative soliton structures," JOSA B **18**, 074005 (2016).

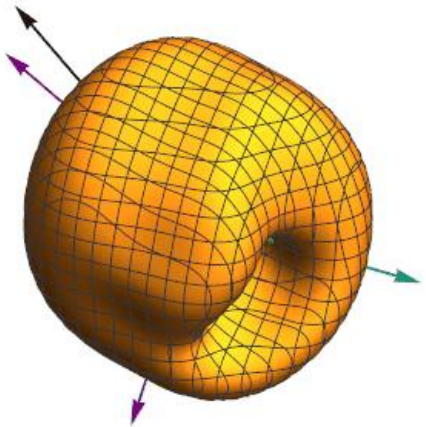
N. A. Veretenov, N. N. Rosanov, and S. V. Fedorov, “Rotating and precessing dissipative-optical-topological-3D solitons,” Phys. Rev. Lett. **117**, 183901 (2016).

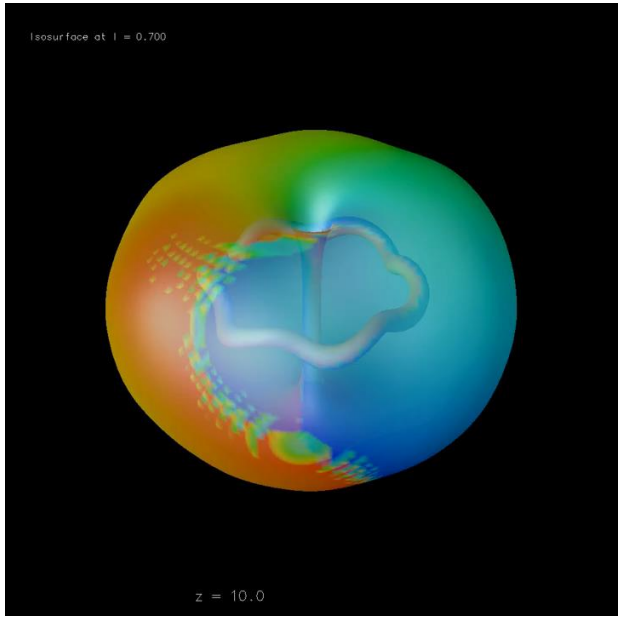
Closed vortex lines (additional to the unclosed one)

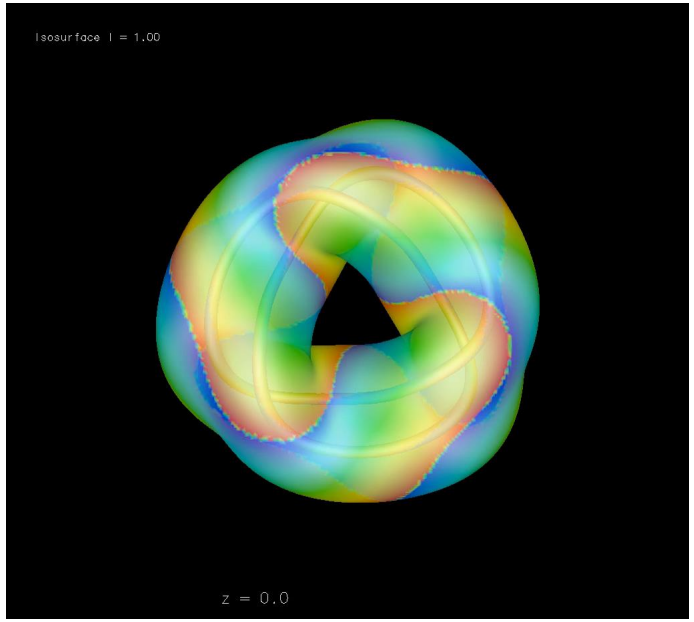
	"Apple", m = 1	Apple-6, m=1	Apple5-6, m=1	2 rings, m=3	Unknot, m=3	Trefoil, m=3
Top view						
Side view						

Common: Unclosed (infinite) vortex line

Dynamic Chicken gods







Chicken god is a stone with a natural hole. The oldest Slavic talisman for protection of cattle and chicken from the evil spirit, including “Kikimora”.

Many thanks to Prof. E.A. Kuznetsov for numerous and helpful discussions. Congratulations and best wishes with his anniversary!

Currently, *Chicken god* brings good luck and health, especially to heroes of the anniversary!