Laboratory and numerical study of intense envelope solitons of water waves

Alexey Slunyaev[1,2], G.F. Clauss[3], M. Klein[3]

[1]Institute of Applied Physics, Nizhny Novgorod, Russia
[2]Nizhny Novgorod State Technical University, Russia
[3]Technical University of Berlin (Germany)

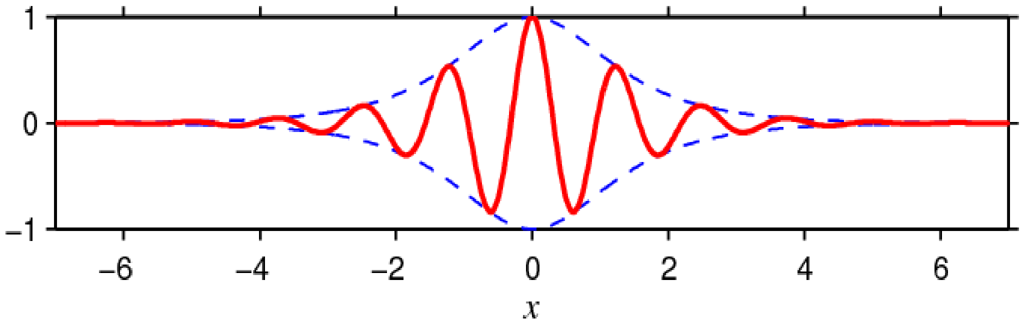
and M. Onorato[4]
[4]Universita di Torino (Italy)

Motivation

Nonlinear Schrodinger equation

Envelope solitons

$$i\left(\frac{\partial A}{\partial t}+\frac{\omega}{2k}\frac{\partial A}{\partial x}\right)+\frac{\omega}{8k^2}\frac{\partial^2 A}{\partial x^2}+\frac{\omega k^2}{2}|A|^2 A=0$$



Equation for modulations
Envelope concept
Envelope soliton

[Zakharov, 1968; Zakharov & Shabat, 1972; AKNS, 1974]

Motivation: Do water wave sol-s exist?

Laboratory simulations

Faster, Steeper, Narrower

Zakharov & Kuznetsov, 1998:

quasisolitons

- Yuen & Lake (Phys Fluids '75):

soliton-like waves for $ka > \sim 0.2$ undergo significant transformation

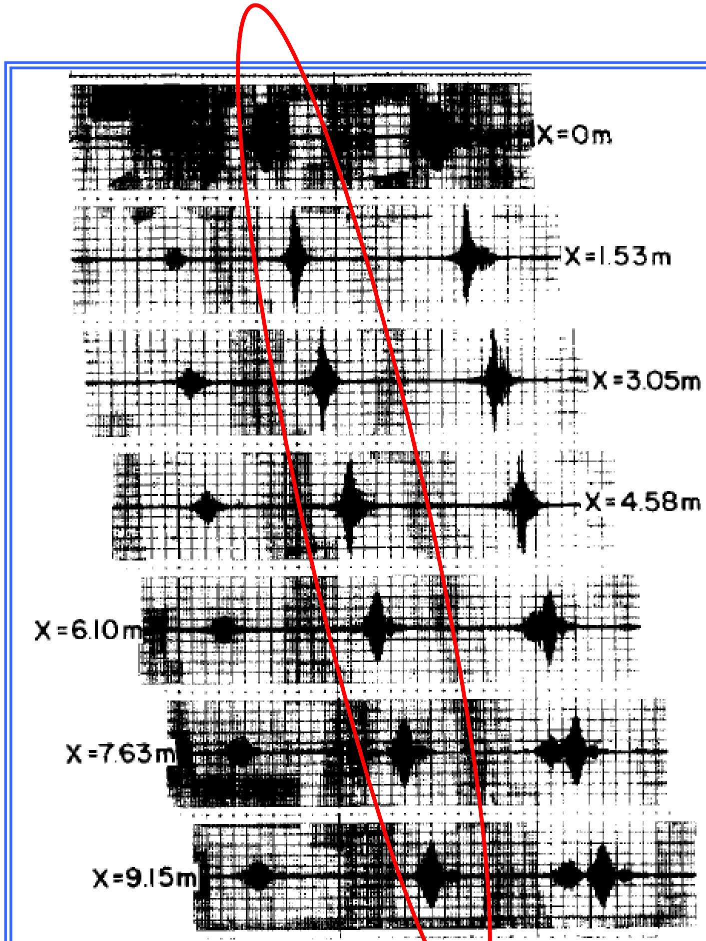


FIG. 3. One wave pulse overtaking and passing through another wave pulse. Left-hand trace: first pulse alone, $\omega_0 = 1.5$ Hz, initial $(ka)_{max} \simeq 0.01$, six-cycle pulse. Center trace: second pulse alone, $\omega_0 = 3$ Hz, initial $(ka)_{max} \simeq 0.2$, 12-cycle pulse which disintegrates into two solitons. Right-hand traces: interaction of the two pulses.

Motivation: Do water wave sol-s exist?

Strongly nonlinear simulations

- Dommermuth & Yue (‘87), West et al (’87) (HOSM)

- Zakharov et al (EJM-B/F ‘06): intense NLS solitons blow up ($ka \sim 0.14$)

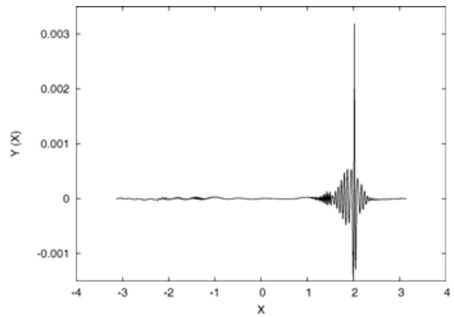


Fig. 24. Surface profile like for *NLSE* soliton with $\mu \simeq 0.14$ at $T = 38.4$.

- Dyachenko & Zakharov (JETPL ‘08): soliton-type groups exist up to wave breaking limit

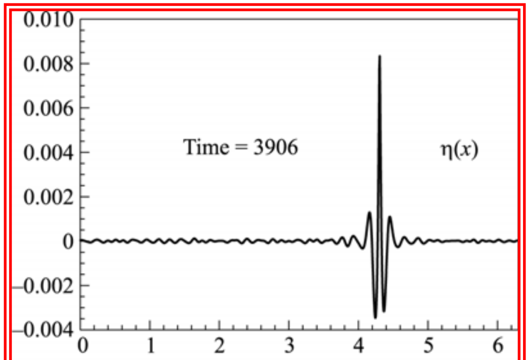
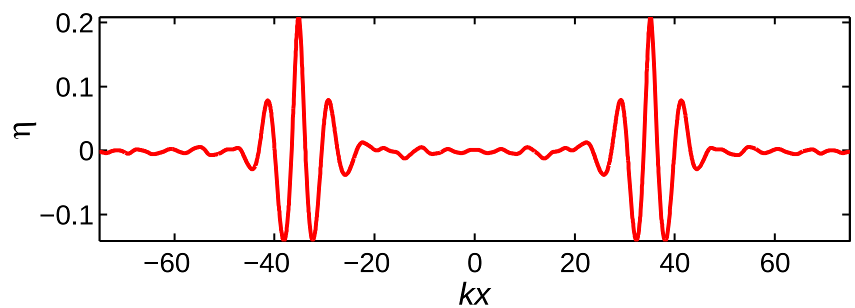


Fig.5. Typical profile of breather

- Slunyaev (JETP ‘09): solitary-like groups exhibit soliton-like behaviour and are relatively well described by envelope equations (up to $ka \sim 0.2$). NLS solitons turn to wave breaking when $ka \sim 0.3$

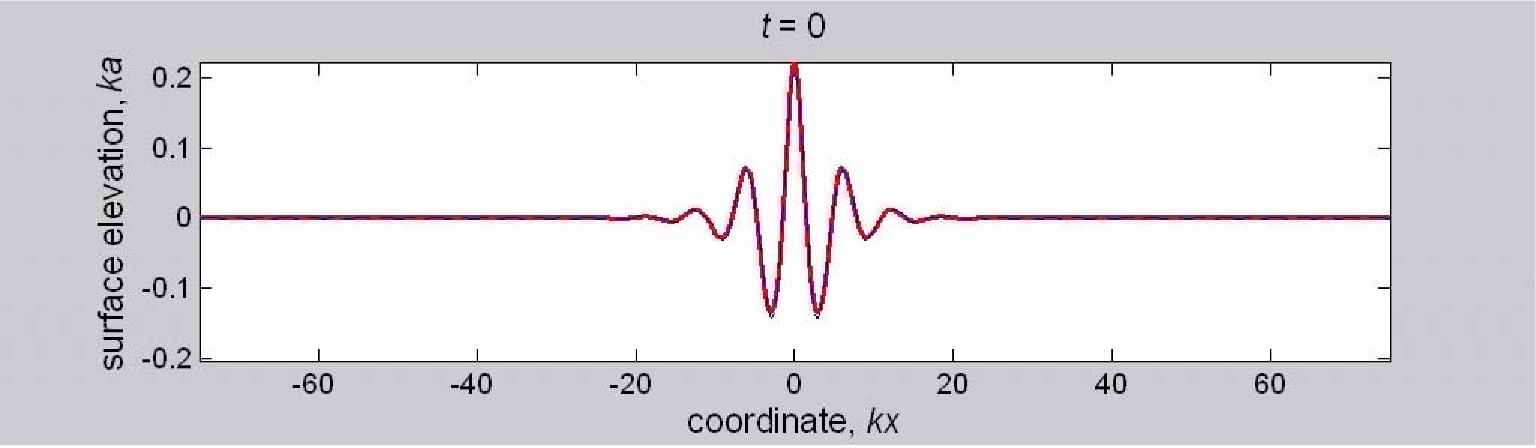
Solitary groups after 6 collisions



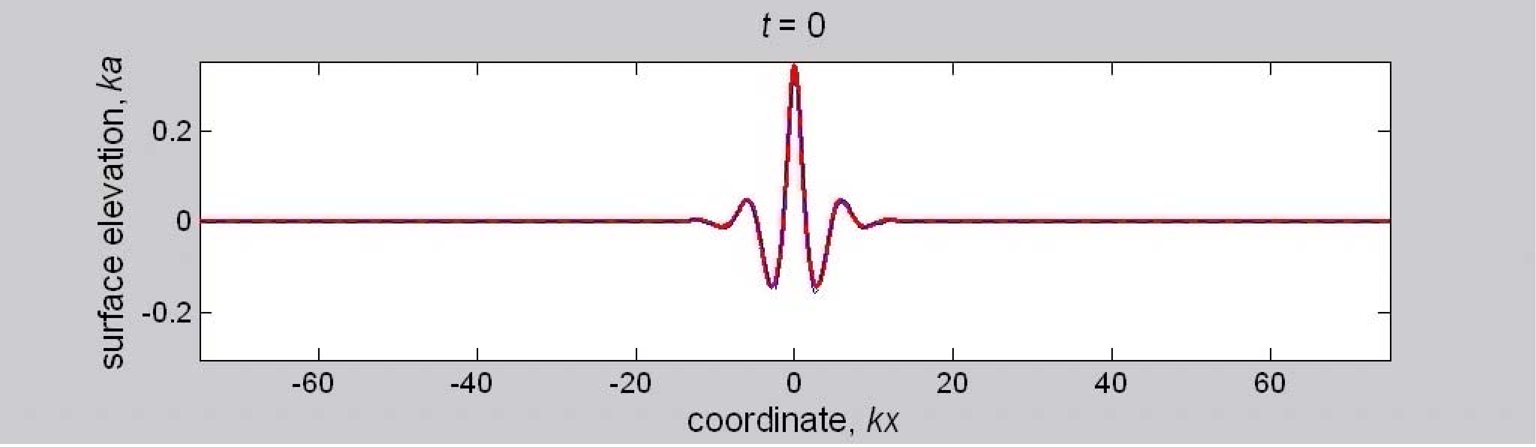
Motivation: Do water wave sol-s exist?

Strongly nonlinear simulations

***ka* = 0.2**



***ka* = 0.3**

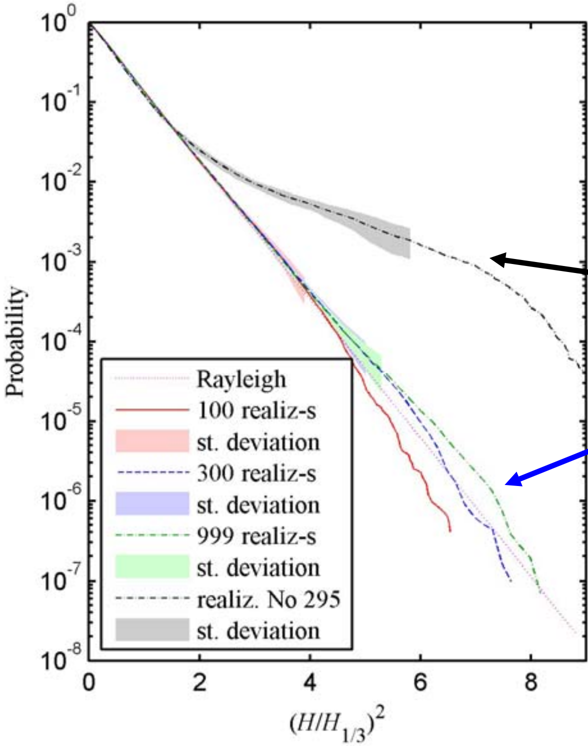


HO NLS eq Euler eqs

Motivation: Rogue waves

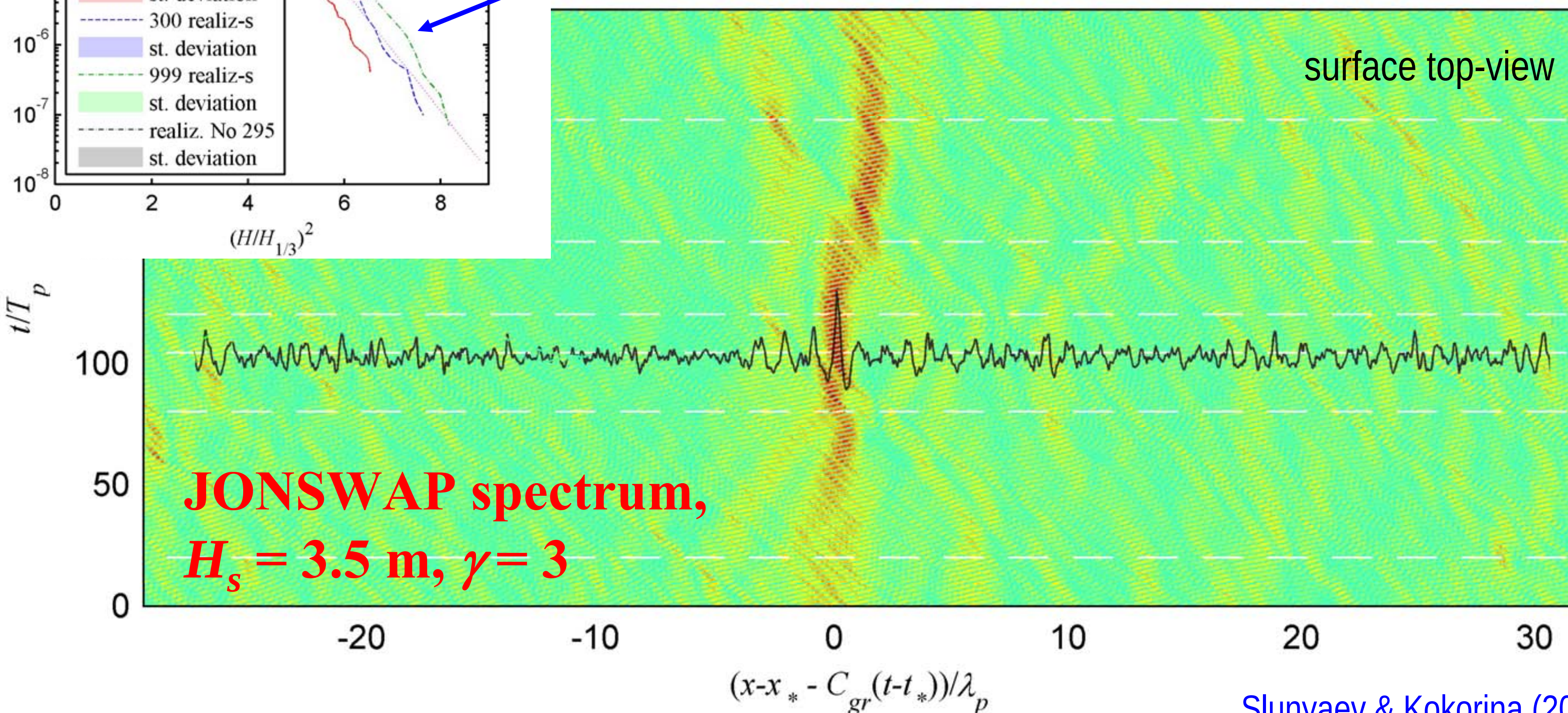
Enormous contribution to the PDF

(potential Euler eqs. framework)



PDF of the realization with an extreme solitary group

Averaged PDF (up to 1000 realizations)

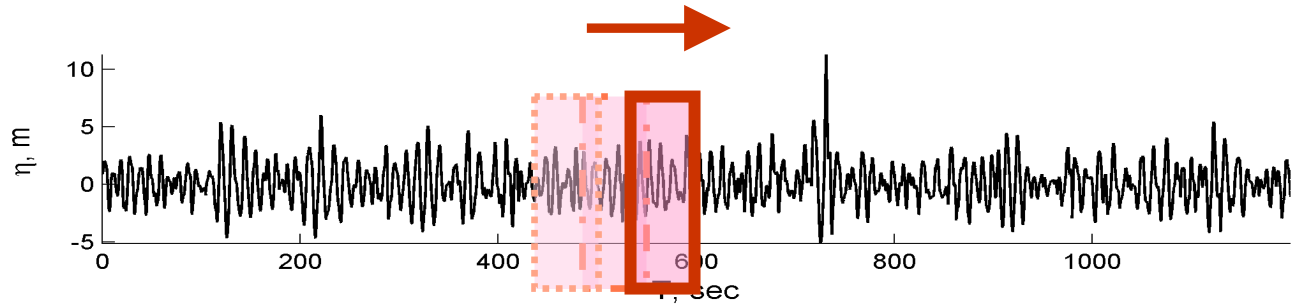


Slunyaev & Kokorina (2017)

Motivation: Rogue waves

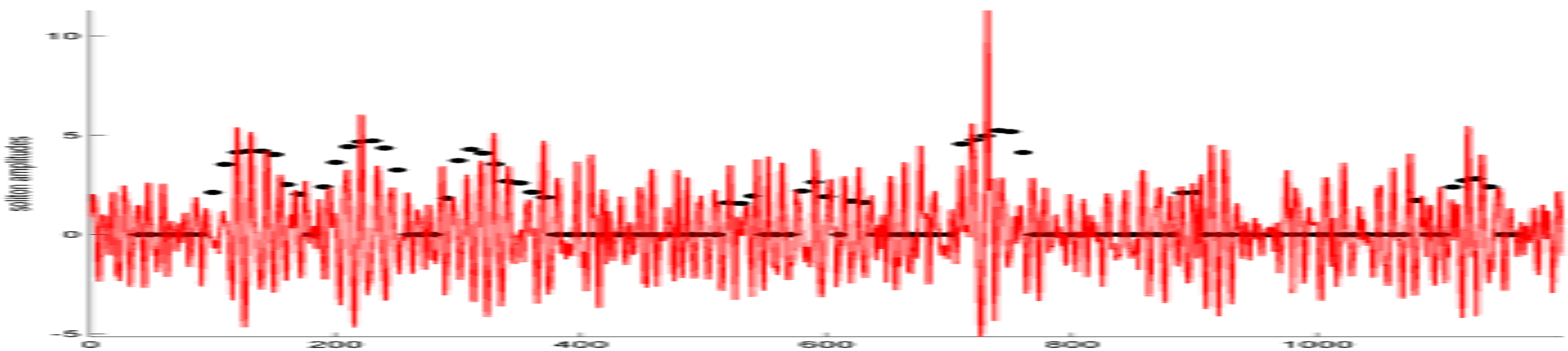
Detection of soliton-like groups in the in-situ records

1. A sliding window extracts samples



2. The scattering problem is solved for the sample implying zero boundary conditions

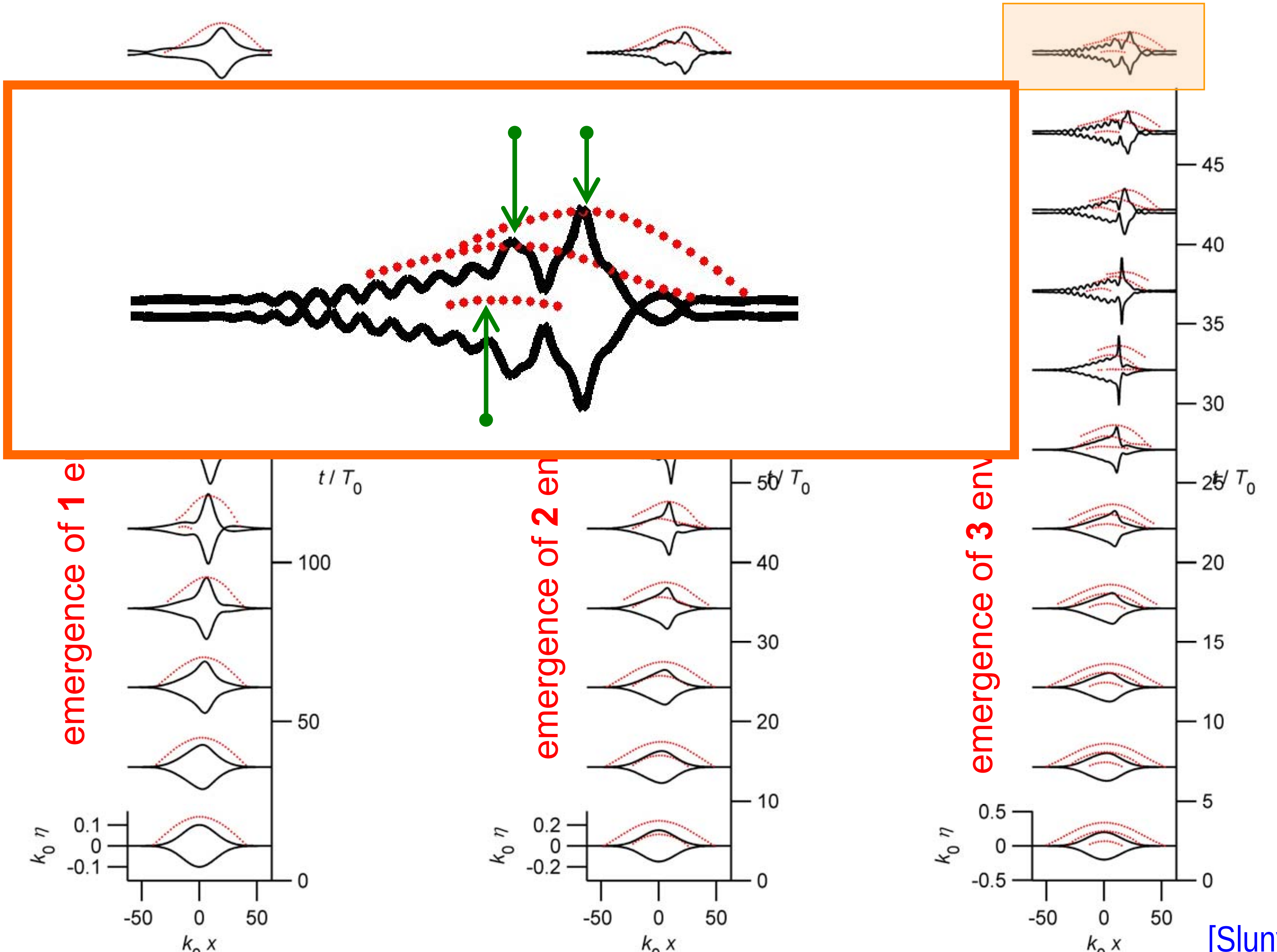
3. The complex eigenvalues describe locally soliton amplitudes and velocities.
   Thus, locations, amplitudes and velocities of the solitons are determined.



4. In terms of the classical Fourier representation, each soliton/breather is a **collective mode**, which retains many Fourier modes coherent.

[Slunyaev et al, 2005, 2006]

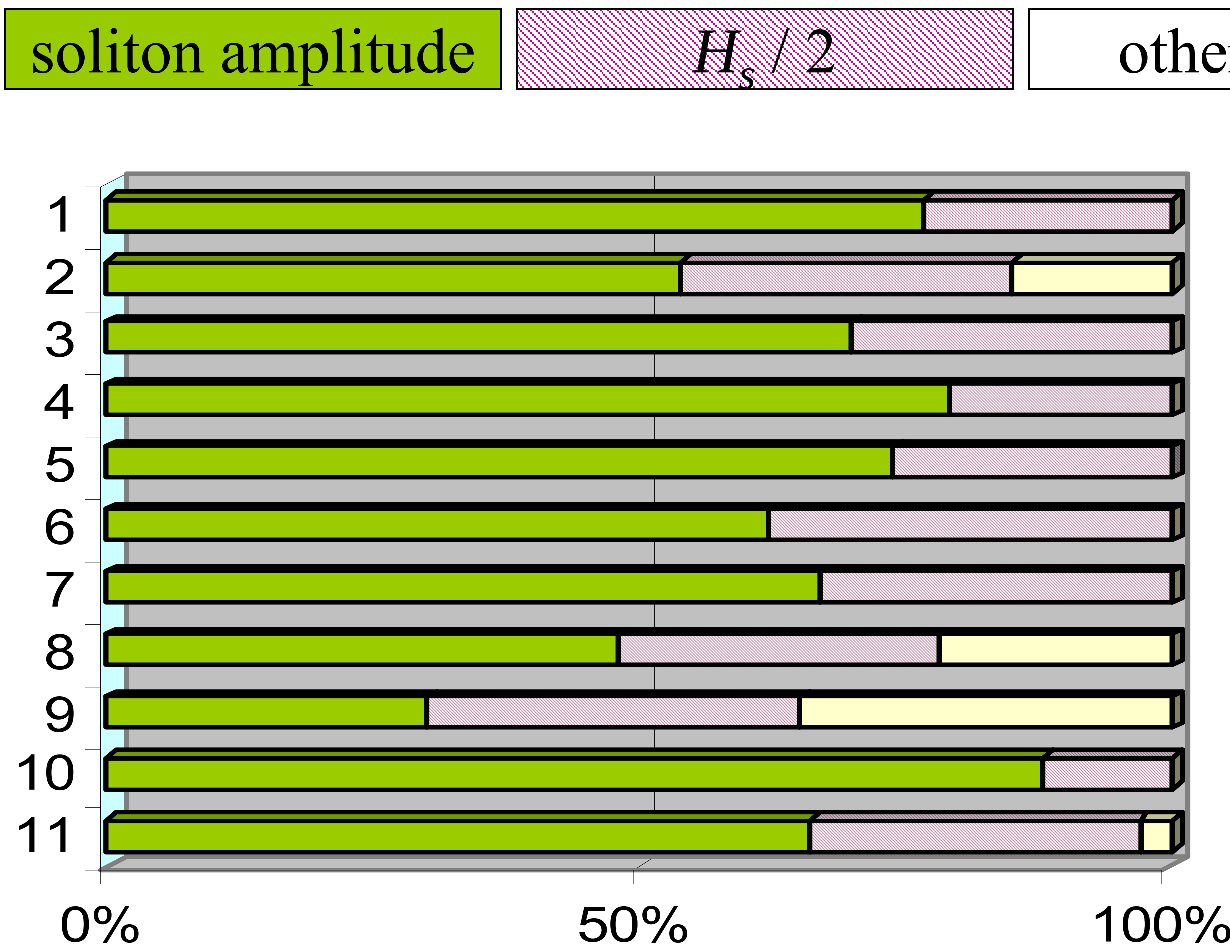
Motivation: Rogue waves



[Slunyaev, 2015]

Motivation: Rogue waves

Detection of soliton-like groups in the in-situ records



records from the North Alwyn platform

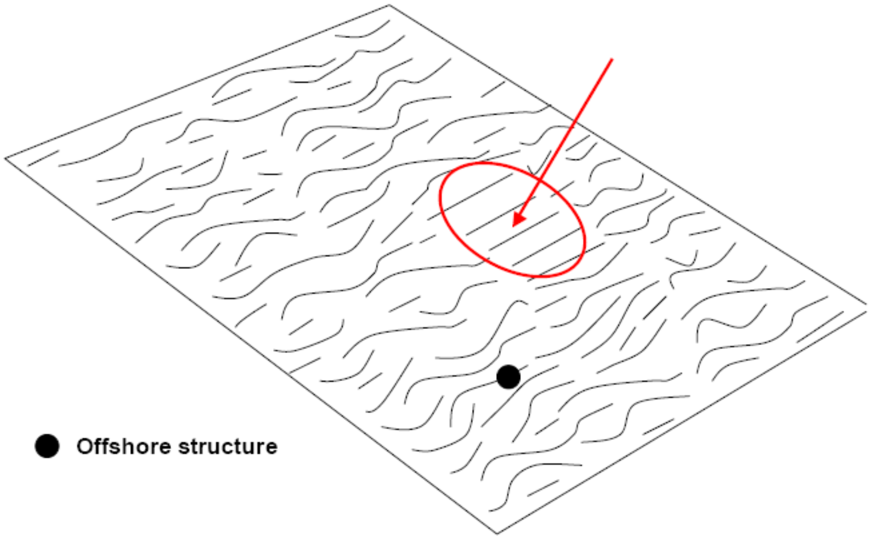
New Year Wave

[Slunyaev, 2006]

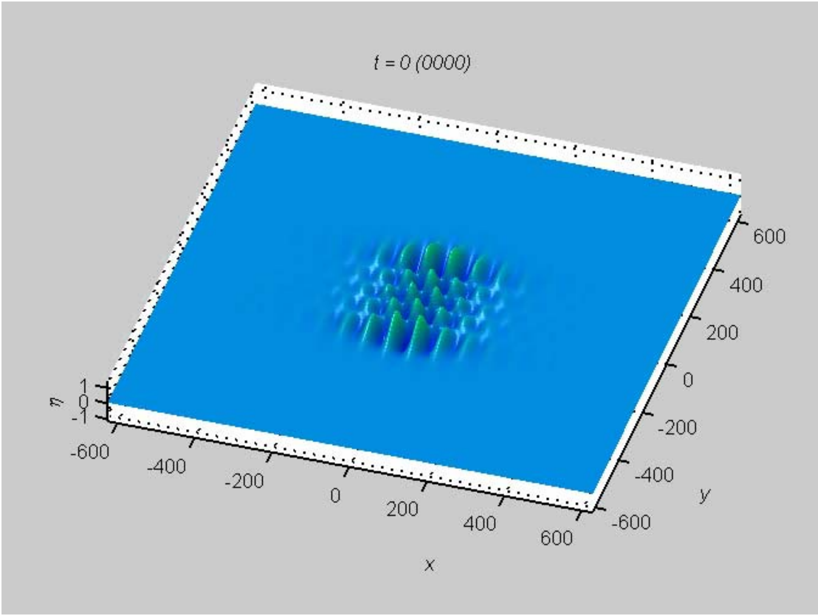
2D solitons in the 3D World

Some possible scenarios

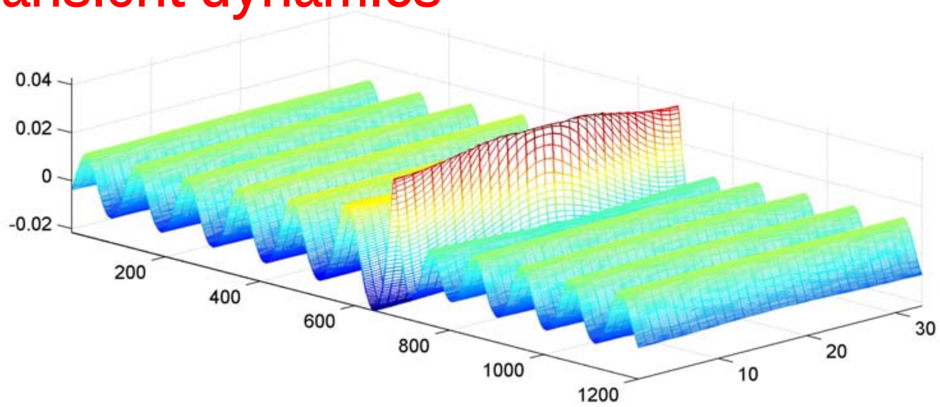
Areas of long-crested wind waves



Trapped wave systems
(by currents, bathymetry, etc)

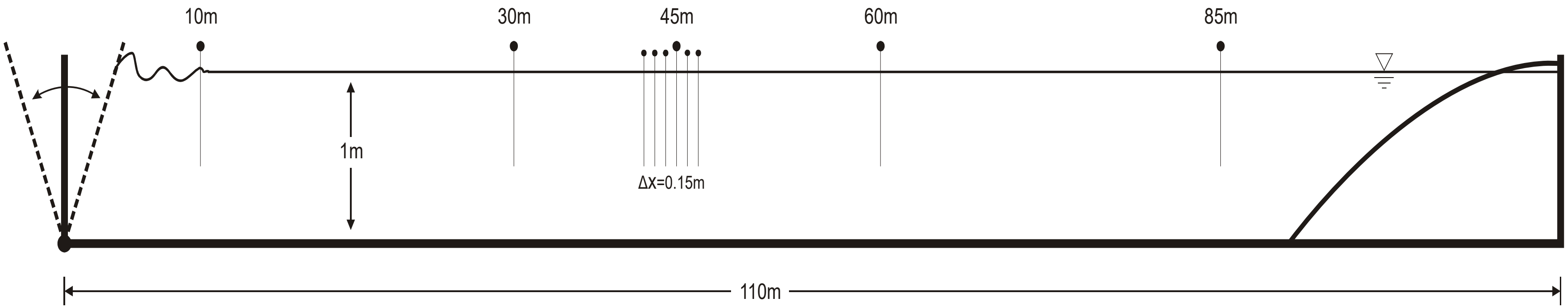


Transient dynamics



I. Single solitons (laboratory)

10 gauges



Various modifications of initial conditions. 43 runs

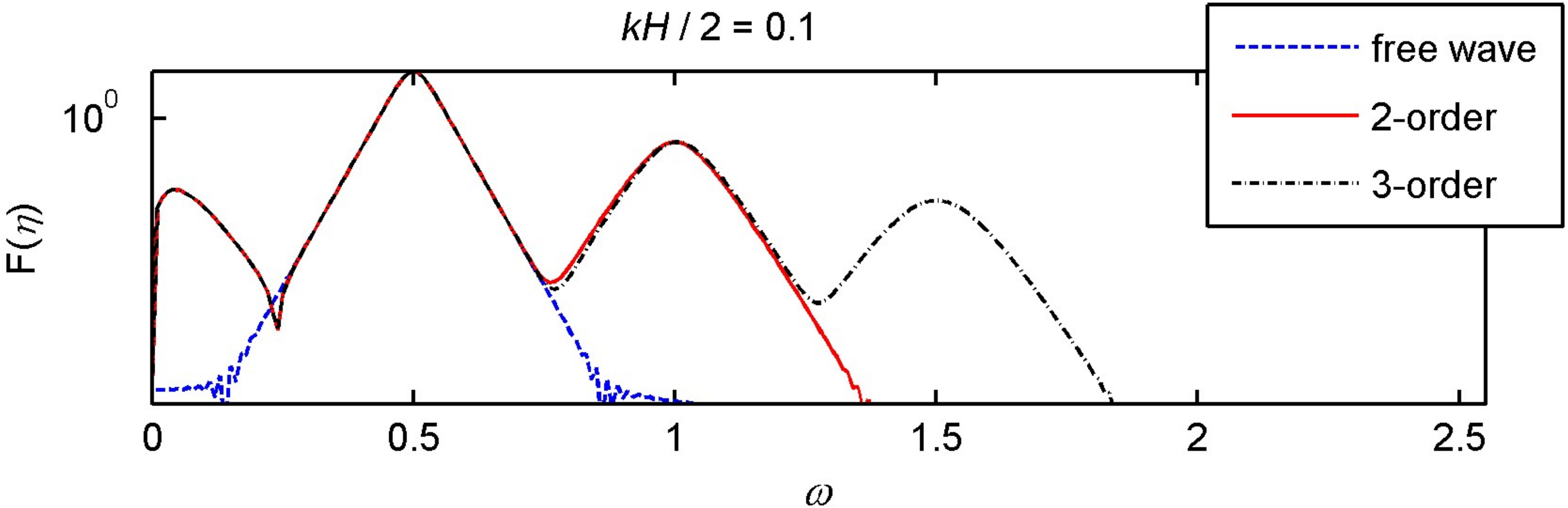
Facility of Technical University of Berlin:

110 meter length (90 m – measuring range)
1 meter depth
8 meter width
PC-controlled wavemaker (**flap**/piston type)
10 gauges

I. Single solitons (laboratory)

How to generate an envelope soliton?

(i) NLS solution (free wave component only)
(ii) NLS solution + 2nd and 3rd harmonics + induced set-down
(ii) Surface elevation from numerical sims of the Euler eqs



Small-amplitude soliton spectrum ($ka = 0.1$)

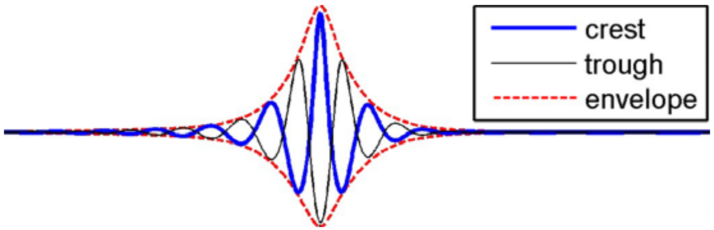
I. Single solitons (laboratory)

How to generate an envelope soliton?

(i) NLS solution (free wave component only)
(ii) NLS solution + 2nd and 3rd harmonics + induced set-down
(ii) Surface elevation from numerical sims of Euler eqs

Approaches (i) and (iii) are the best

(a) Group with high crest
(b) Group with deep trough



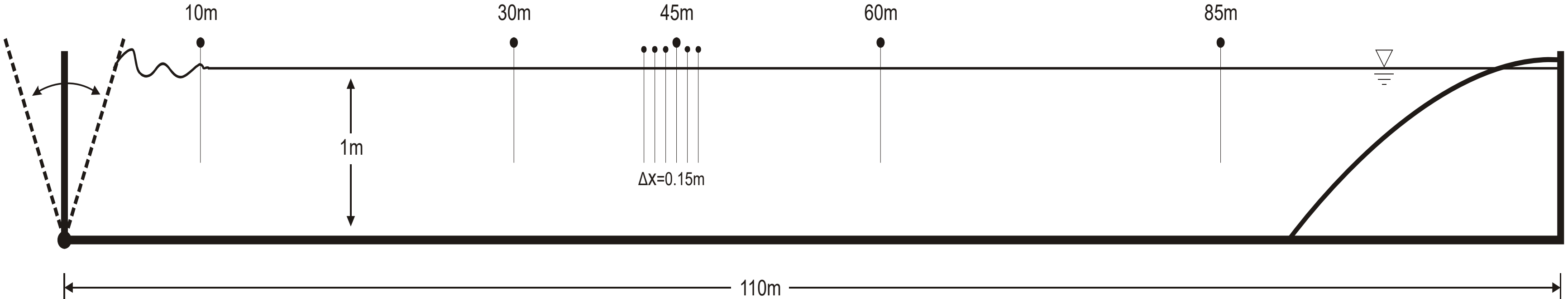
No difference

(∗) Different carrier frequencies (depths k_0h = 3.57…5.76)

The results are sensitive to the carrier frequency

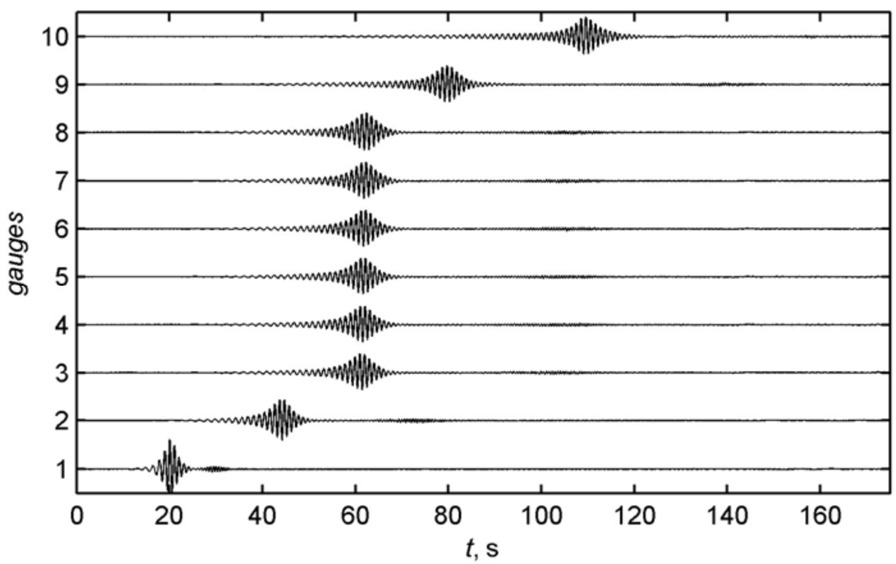
I. Single solitons (laboratory)

10 gauges

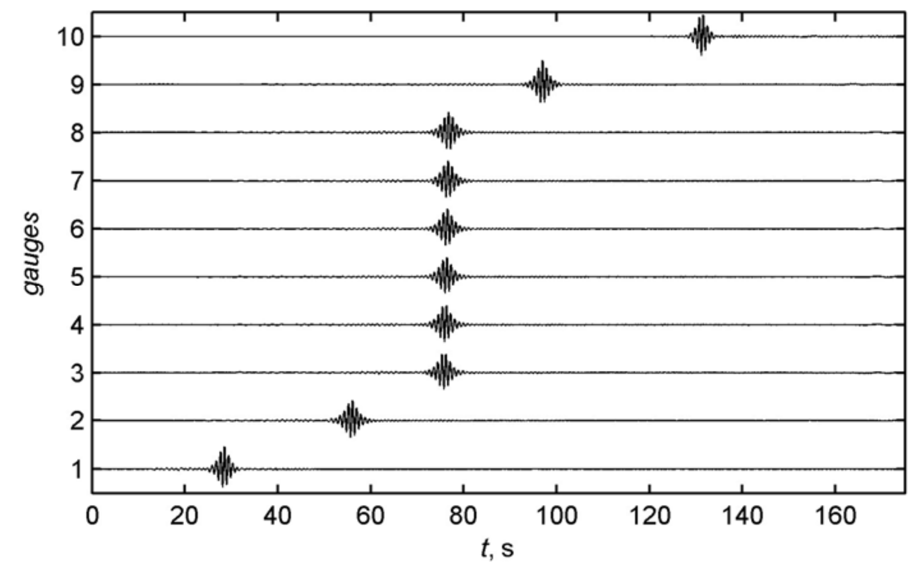


Various modifications of initial conditions. 43 runs

Examples of records



$kA \approx 0.3$!



I. Single solitons (numerical)

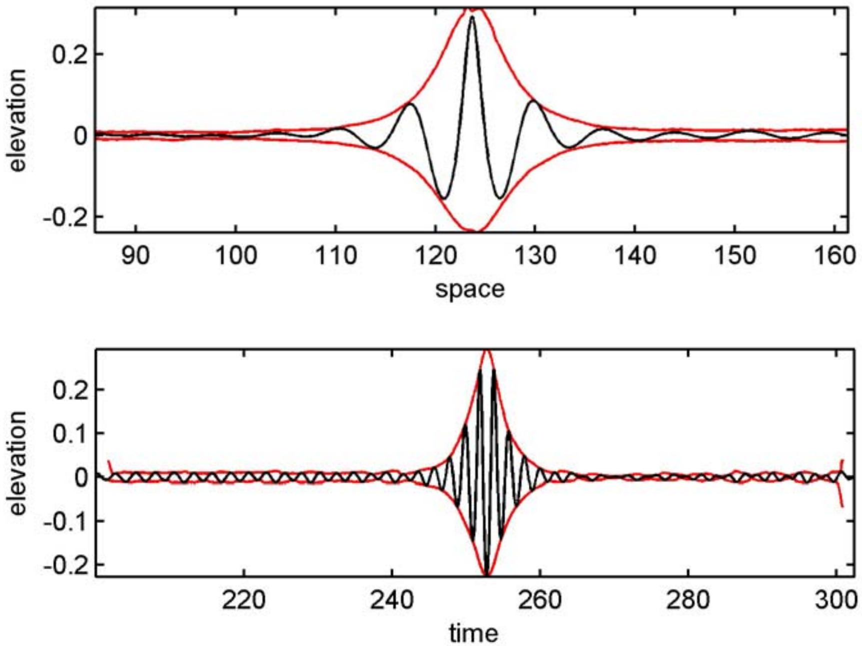
Initial condition – envelope soliton NLS + induced current + 2nd and 3rd harmonics

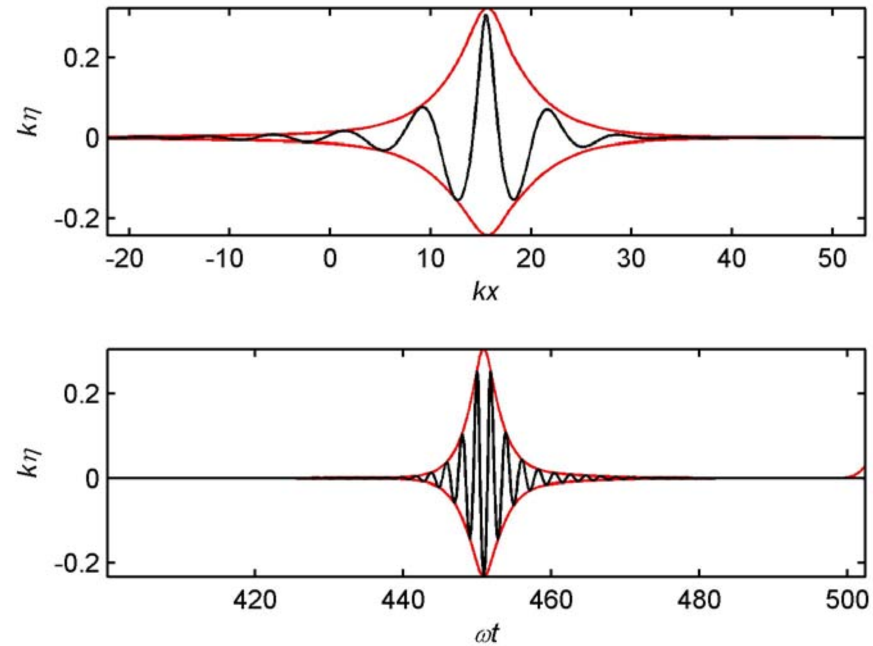
Model – HOSM (*M* = 6) / Euler eqs

Output: space series and time series

Damping mask to filter out the radiation

***ka* ≈ 0.3**

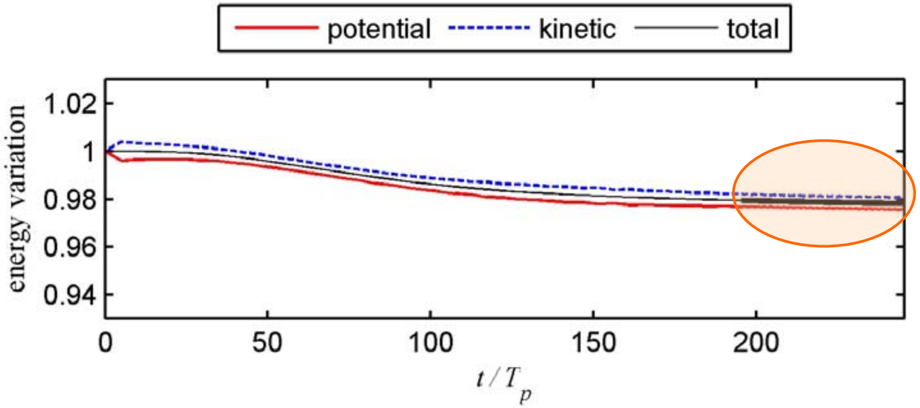


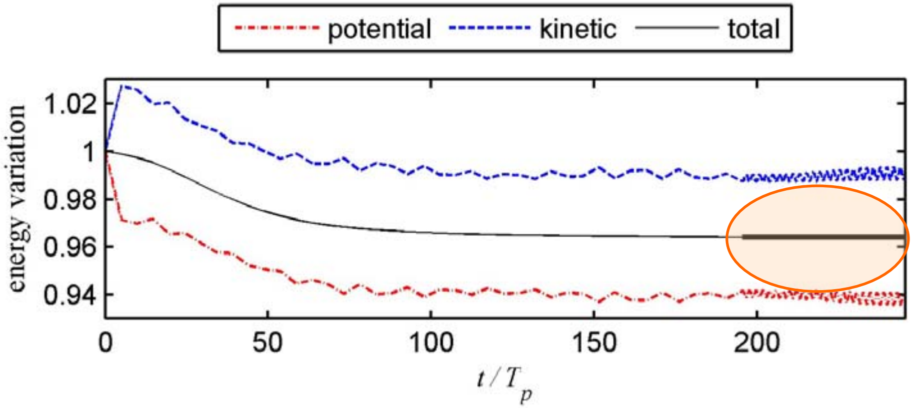


I. Single solitons (numerical)

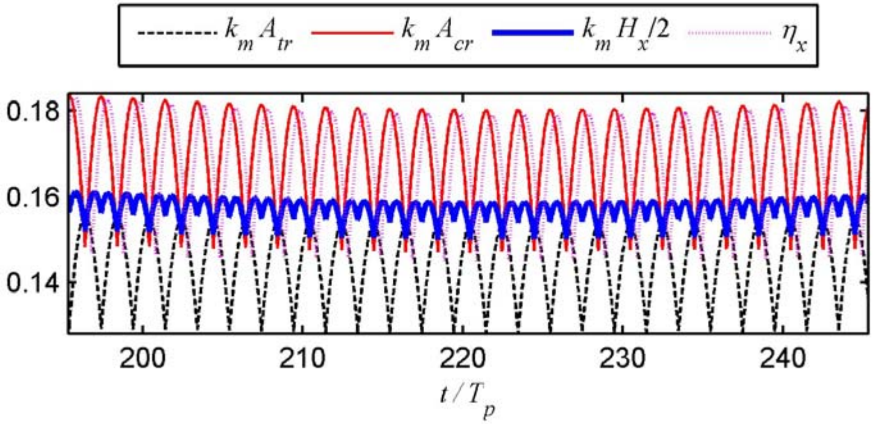
case $k_0 a \approx 0.2$ | case $k_0 a \approx 0.3$

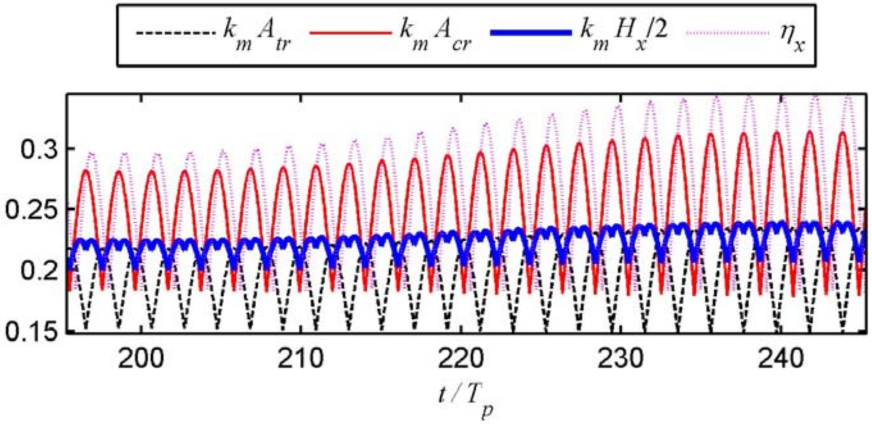
Energy





Wave intensity

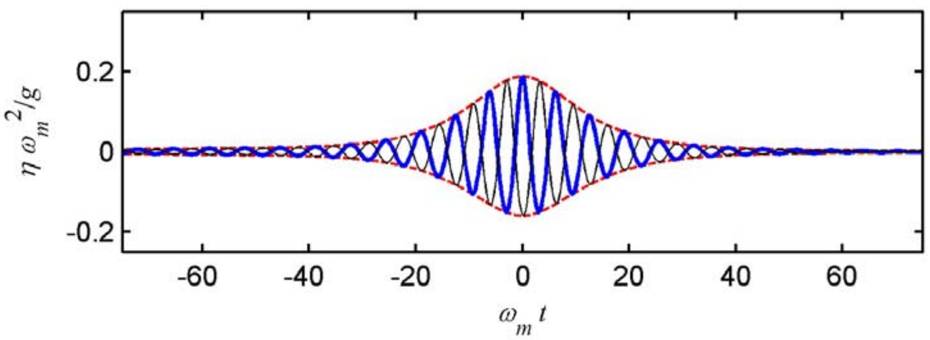


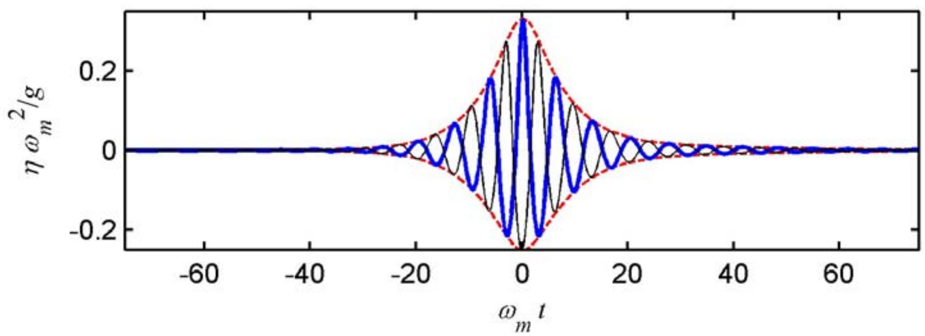


I. Single solitons (numerical)

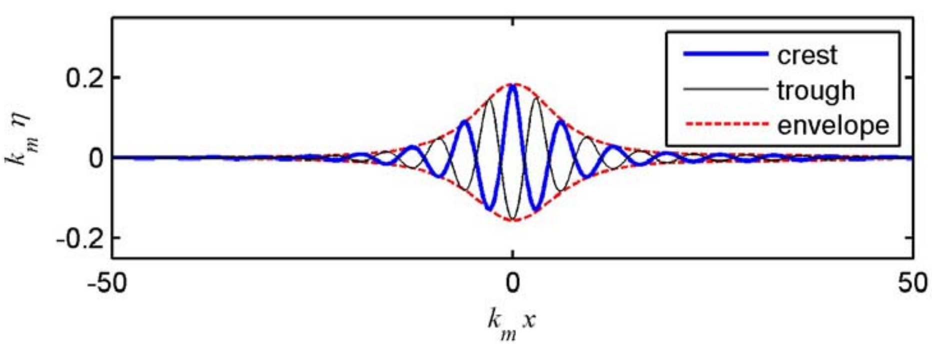
case $k_0 a \approx 0.2$ | case $k_0 a \approx 0.3$

Time series

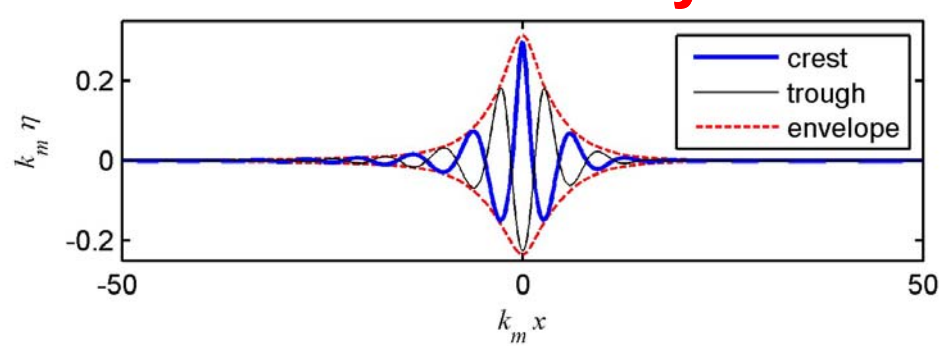




Snapshots



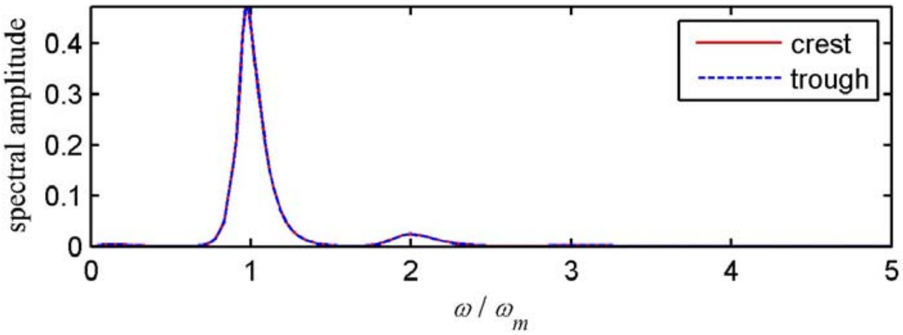
Just a few wave cycles!

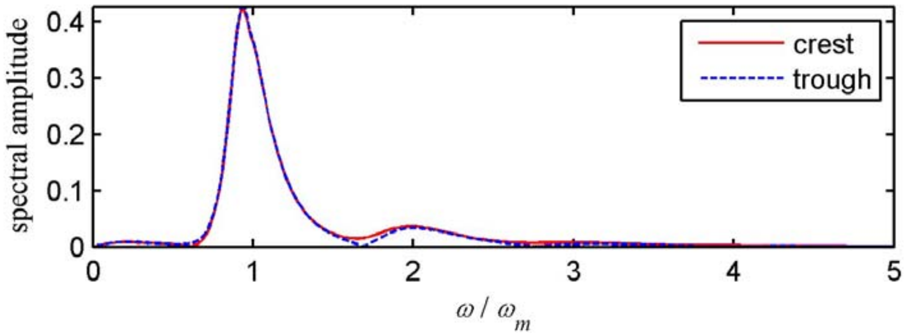


I. Single solitons (numerical)

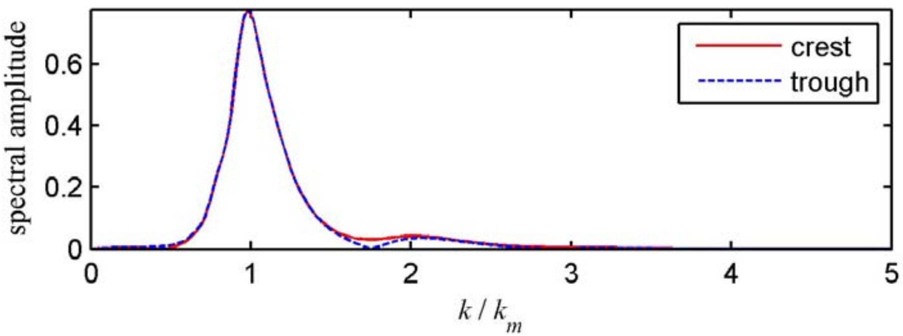
case $k_0 a \approx 0.2$ | case $k_0 a \approx 0.3$

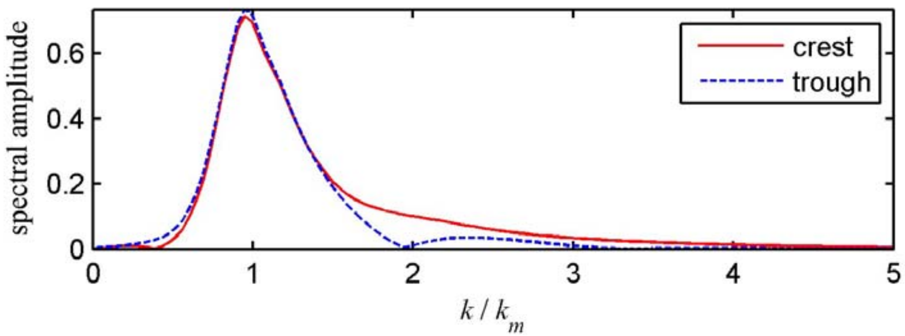
Frequency Fourier transform





Wavenumber Fourier transform

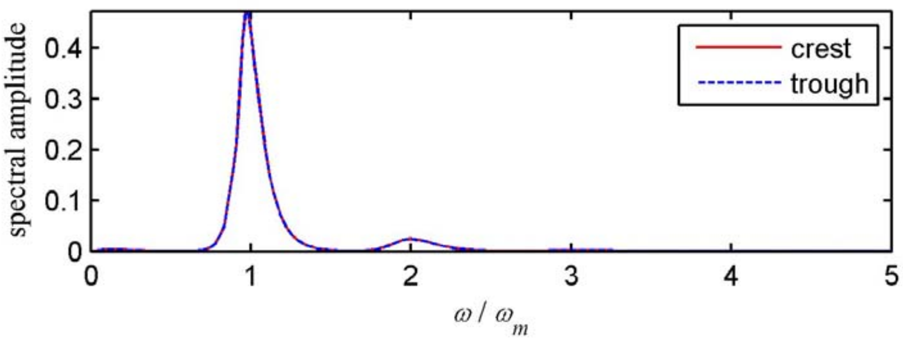


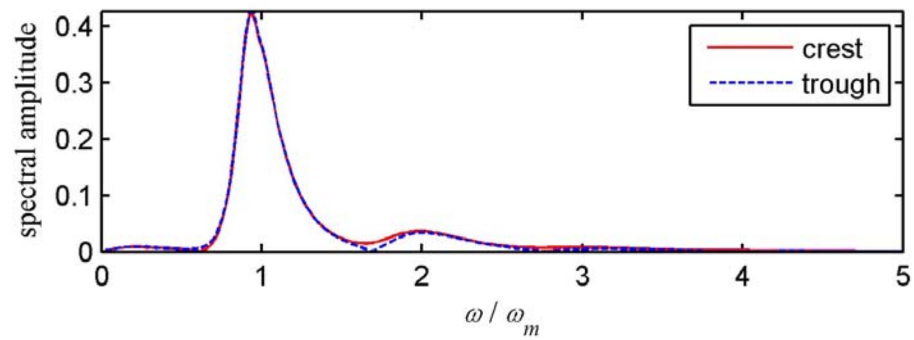


I. Single solitons (numerical)

case $k_0 a \approx 0.2$ | case $k_0 a \approx 0.3$

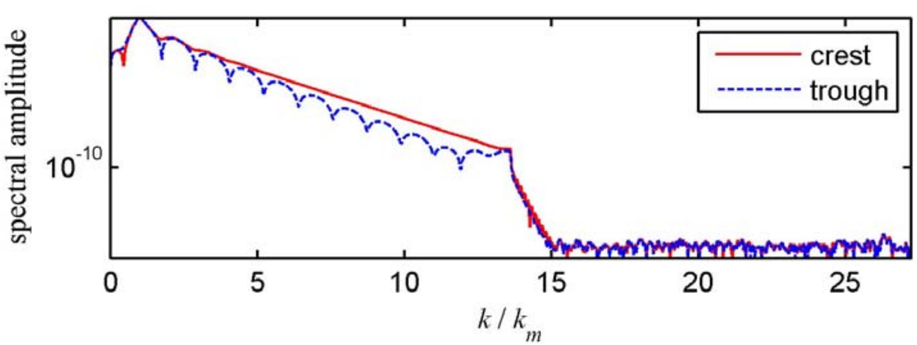
Frequency Fourier transform

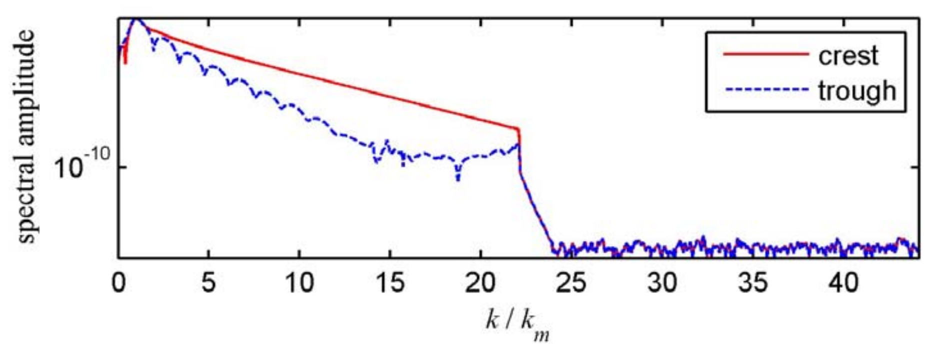




Wavenumber Fourier transform (semi-log scales)

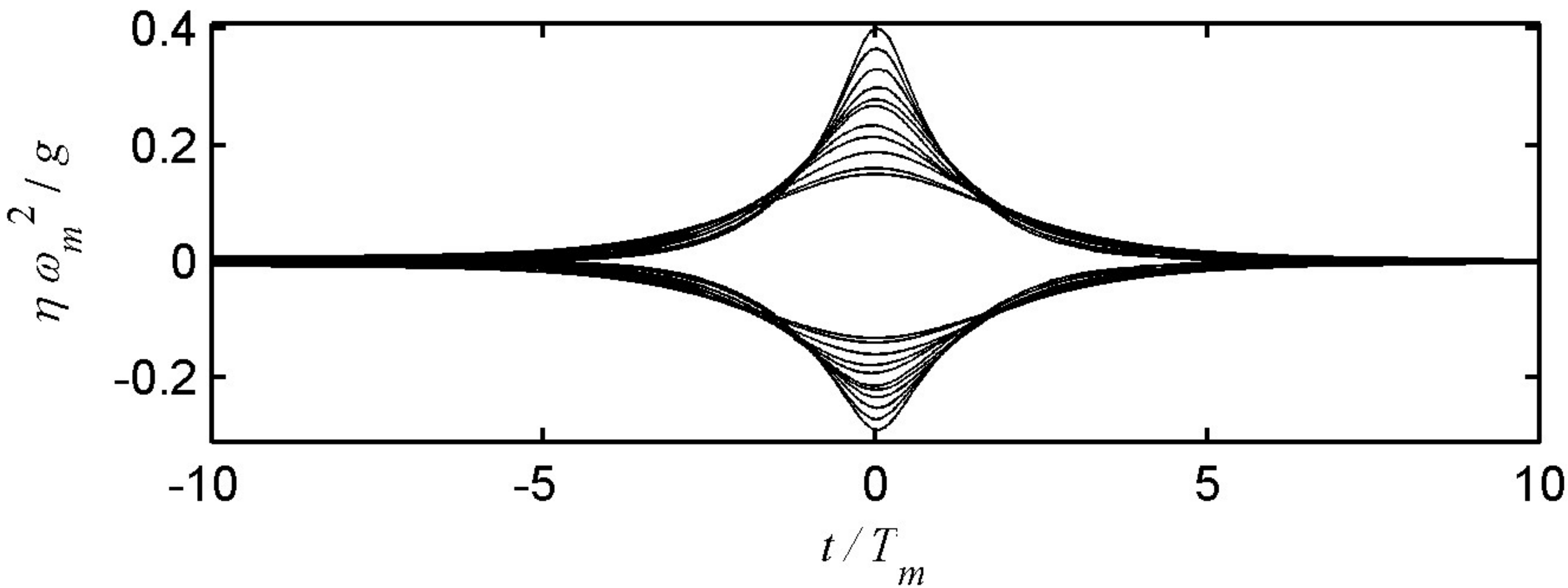
Broad spectrum





I. Single solitons (numerical)

Envelope soliton shapes for different steepness

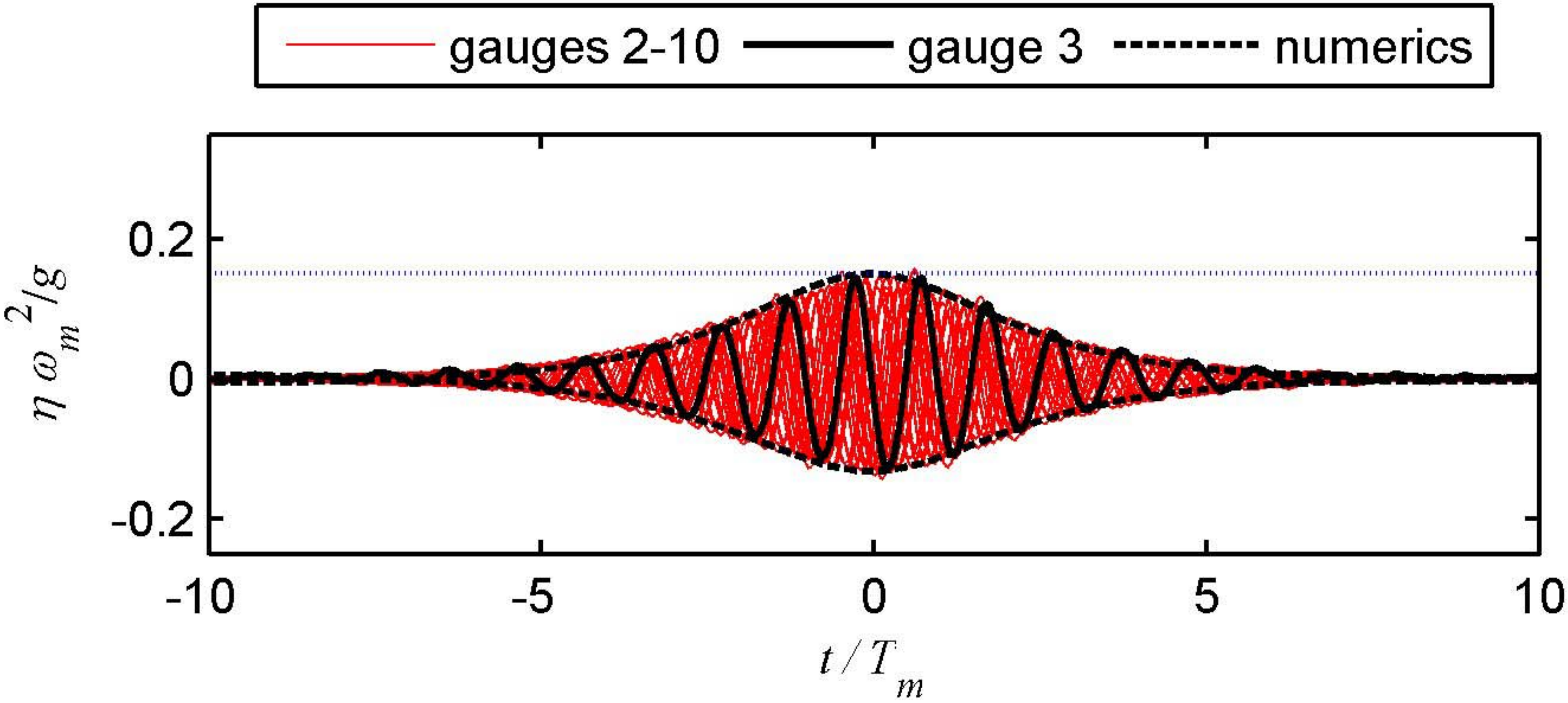


I. Single solitons (lab. vs num.)

Surface displacements from lab measurements

and envelopes from numerical runs

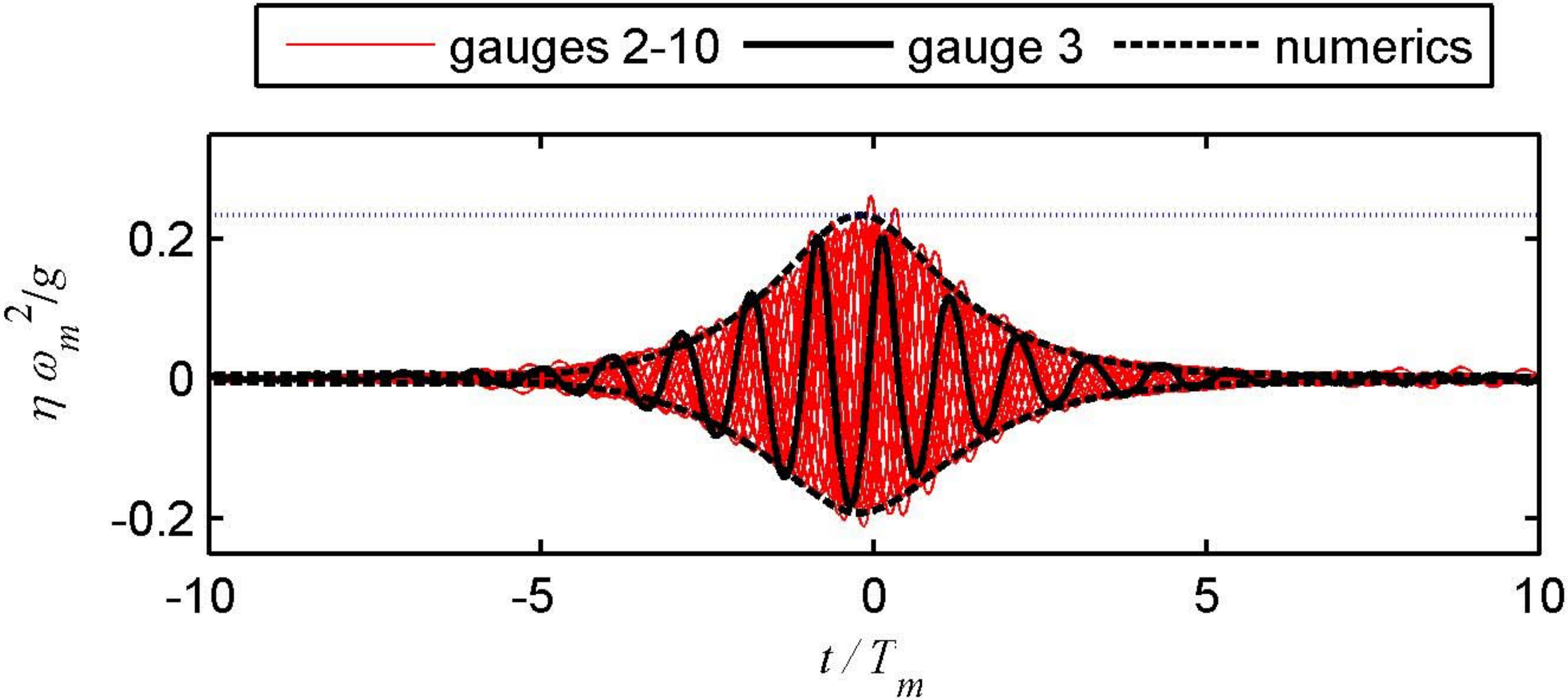
$A_{cr}\omega^2_m/g = 0.150$



I. Single solitons (lab. vs num.)

Surface displacements from lab measurements and envelopes from numerical runs

$A_{cr}\omega^2_m/g = 0.235$

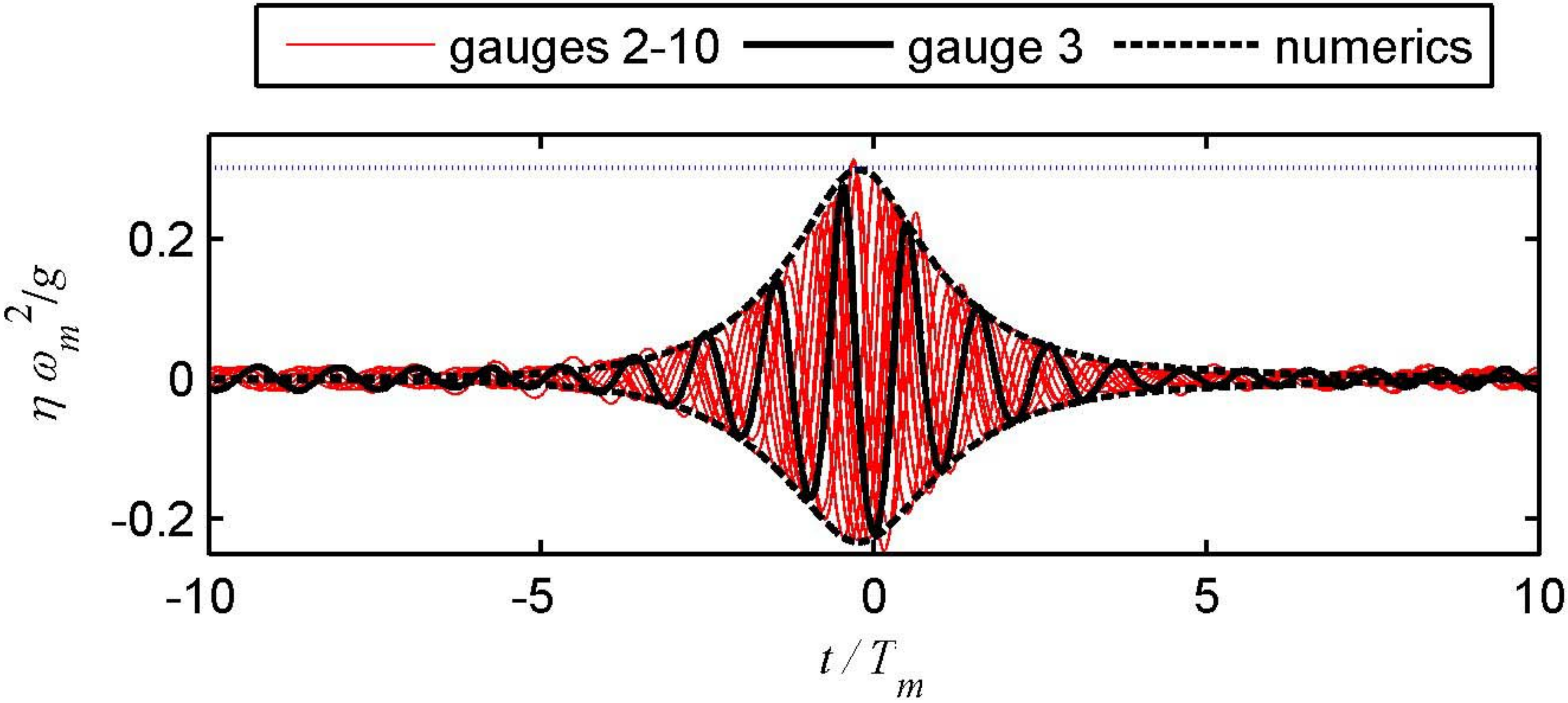


I. Single solitons (lab. vs num.)

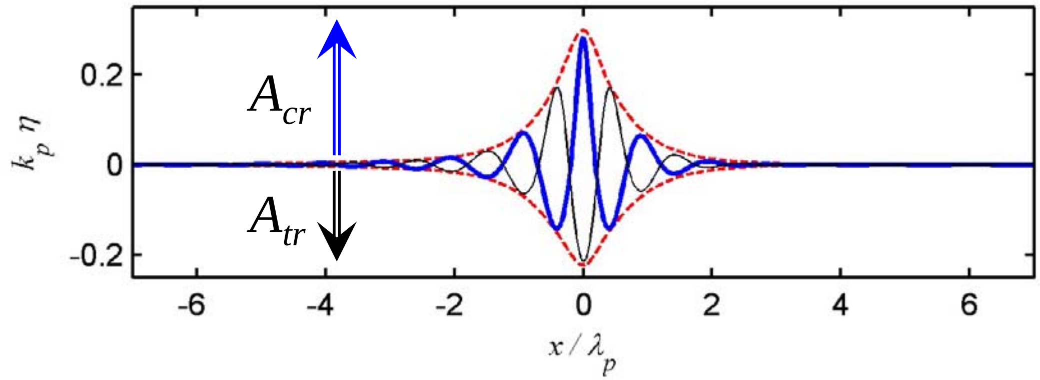
Surface displacements from lab measurements

and envelopes from numerical runs

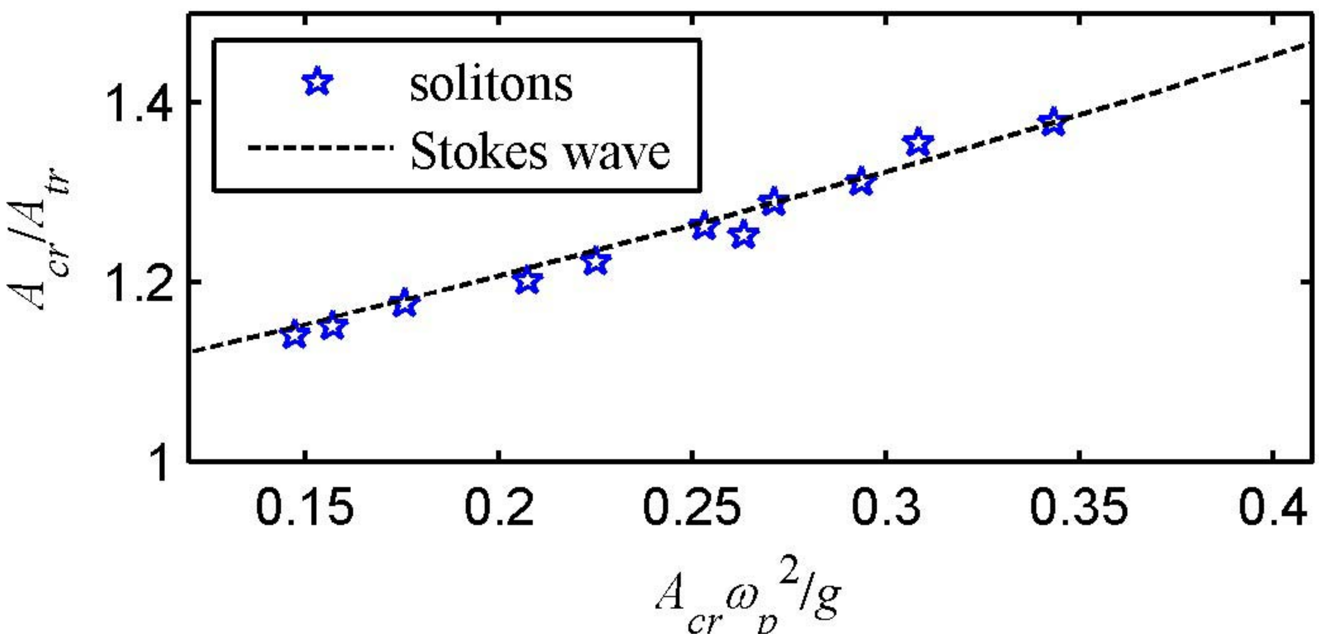
$A_{cr}\omega^2{}_m/g = 0.301$



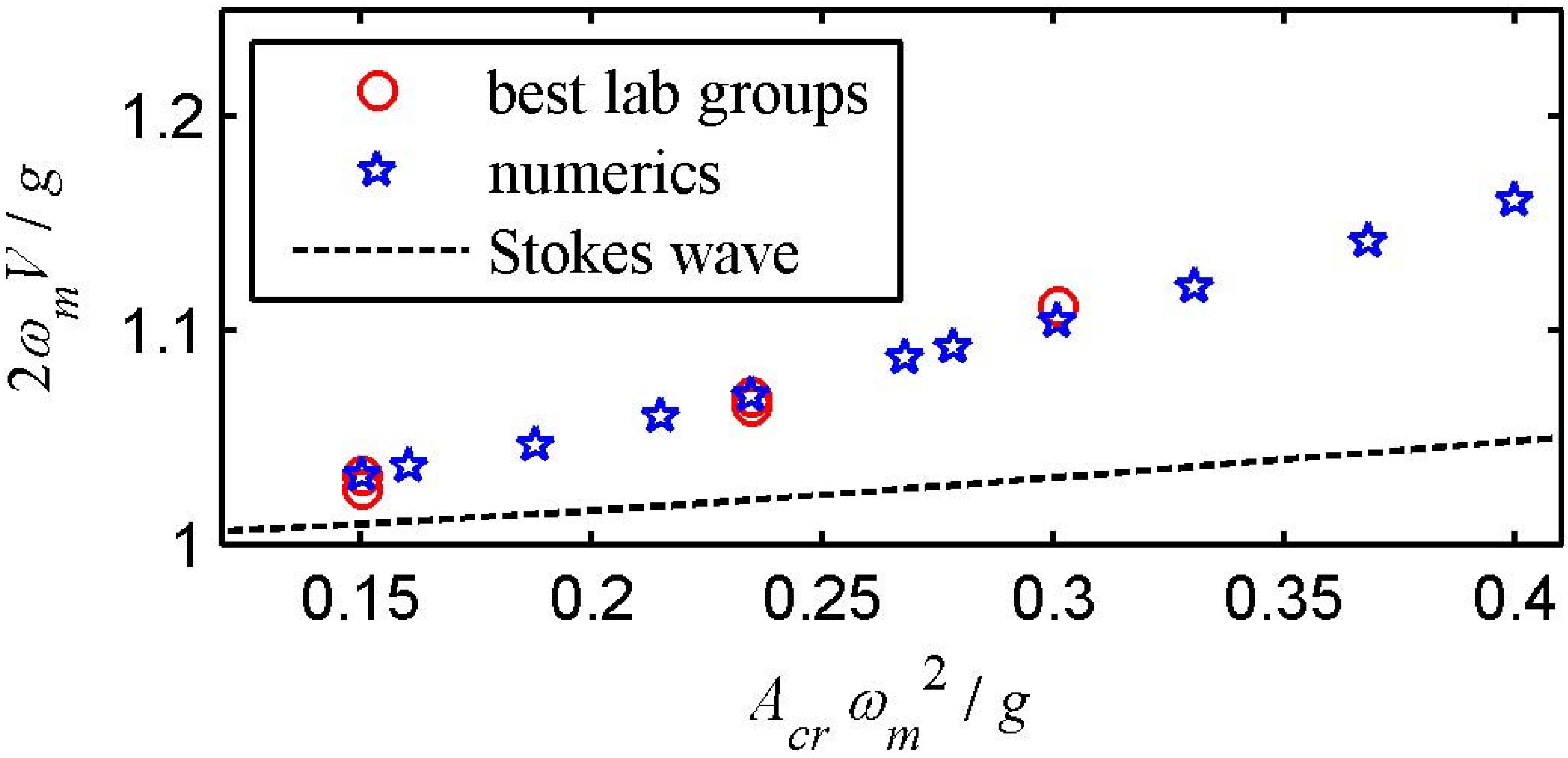
Wave asymmetry [results from num sims]



Wave asymmetry of the nonlinear wave groups is similar to the one of Stokes' waves



Wave velocity [lab. vs num.]



The effect of nonlinearity on the wave group velocity is significantly greater than in the case of regular Stokes waves

Conclusions (intermediate)

Steep and short solitary waves with the steepness up to $A_{cr}\omega^2_m/g$ = 0.3 are able to pass at least 60 wavelengths (15-30 group lengths). Steeper initial conditions experience local breaking.
The soliton groups exist in the whole range of wave steepnesses*.

* probably limited by 3D effects

Very good agreement between the laboratory measurements and strongly nonlinear simulations of the potential eq-s is observed.
The NLS solution may be reasonably accurate for some purposes

The solitary groups exhibit vertical asymmetry similar to the Stokes waves case. Thus, having a snapshot it is difficult/impossible to say if a wave belongs to a soliton group

The nonlinear speed-up of the solitary groups is even more significant than in the case of Stokes waves. Solitons move faster

A. Slunyaev, G.F. Clauss, M. Klein, M. Onorato, Phys. Fluids 25, 067105 (2013)

II. Soliton collisions

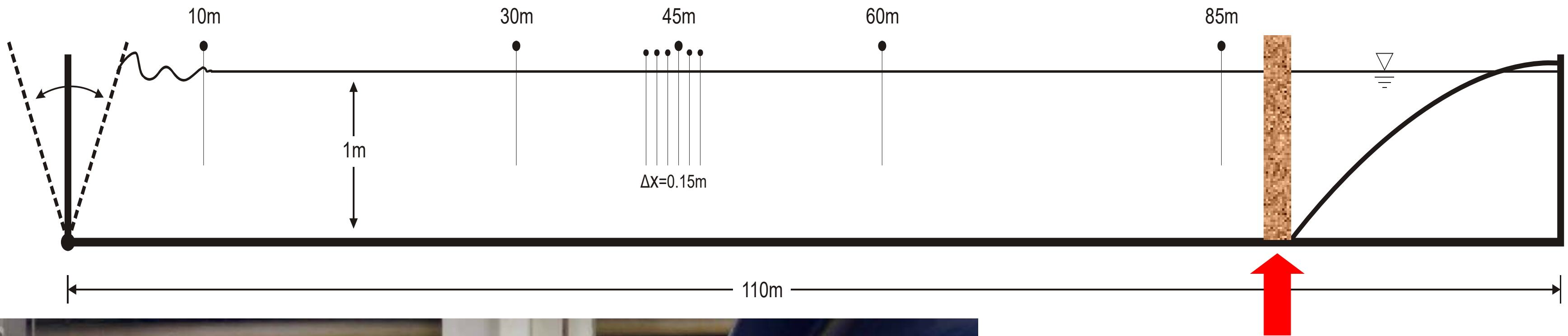
1. Reflection from the vertical wall

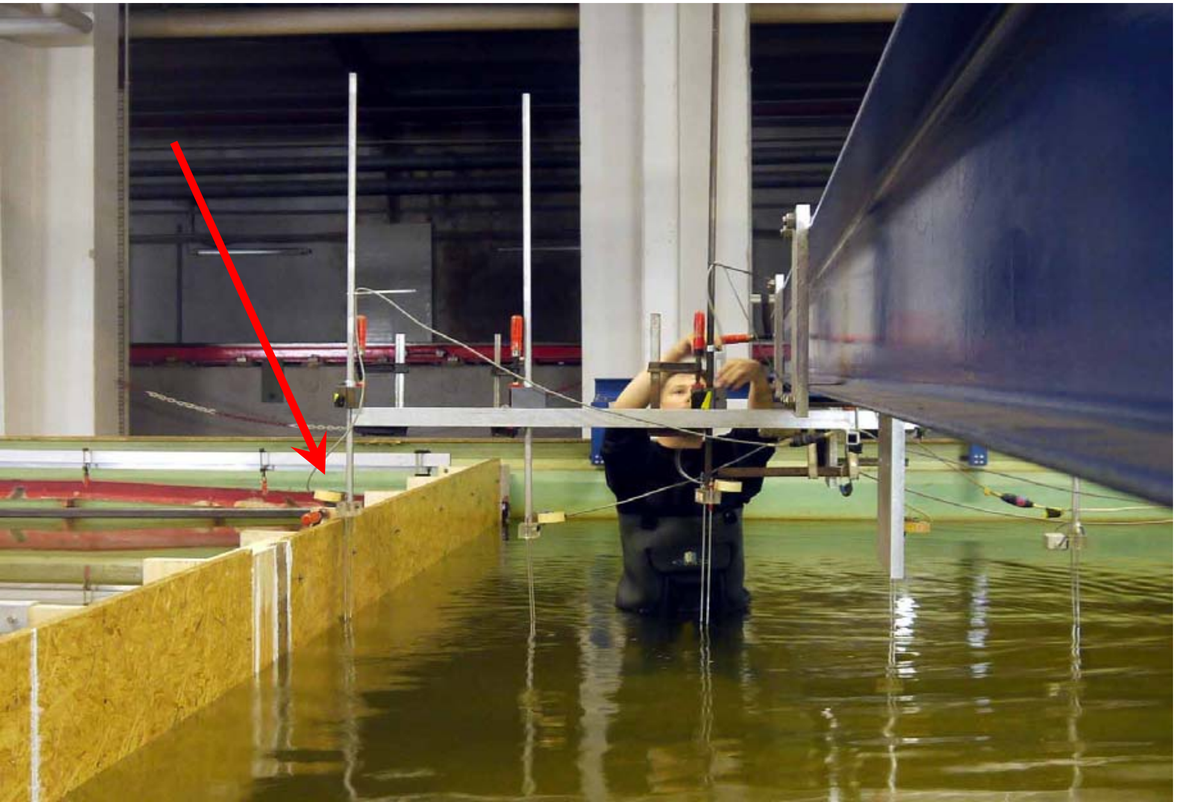
2. Head-on collisions

3. Overtaking interactions

II. Soliton collisions (laboratory)

9 gauges, different locations



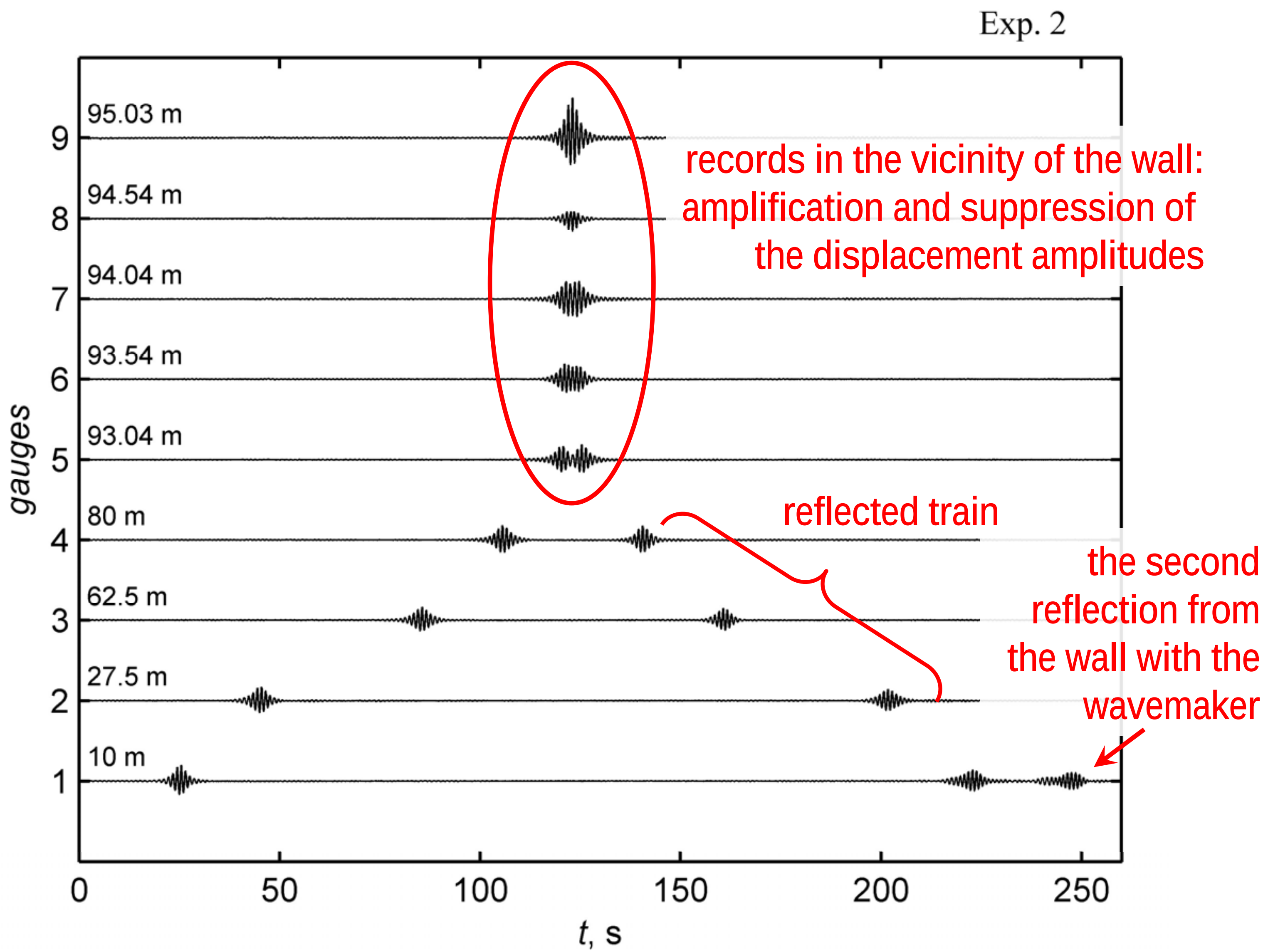


A rigid wall was mounted made of oriented strand boards braced with wooden beams

95 meter long
1 meter depth
8 meter width
9 gauges
30 runs

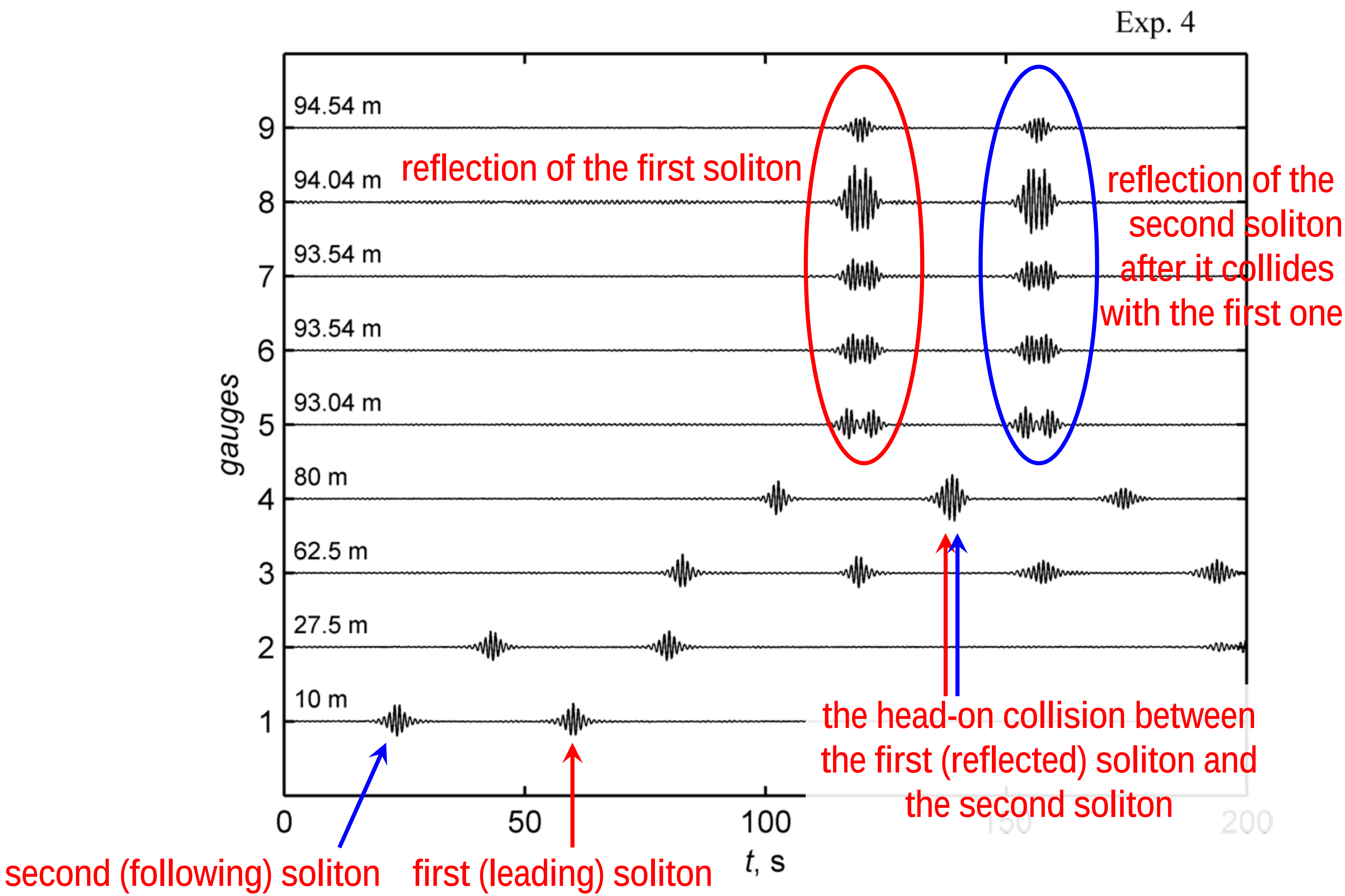
II. Soliton collisions (laboratory)

Reflection from the wall



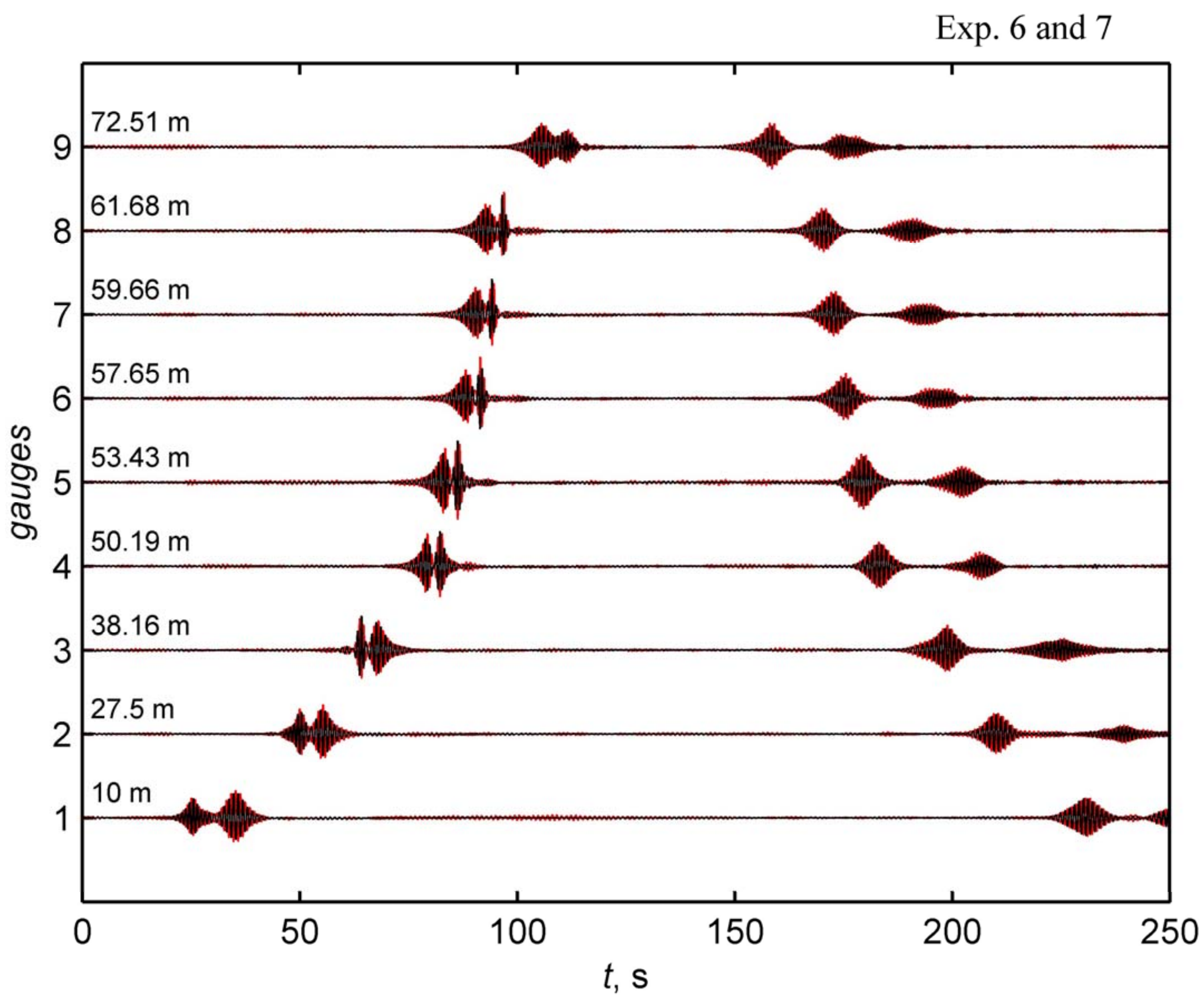
II. Soliton collisions (laboratory)

Reflection from the wall and head-on collision

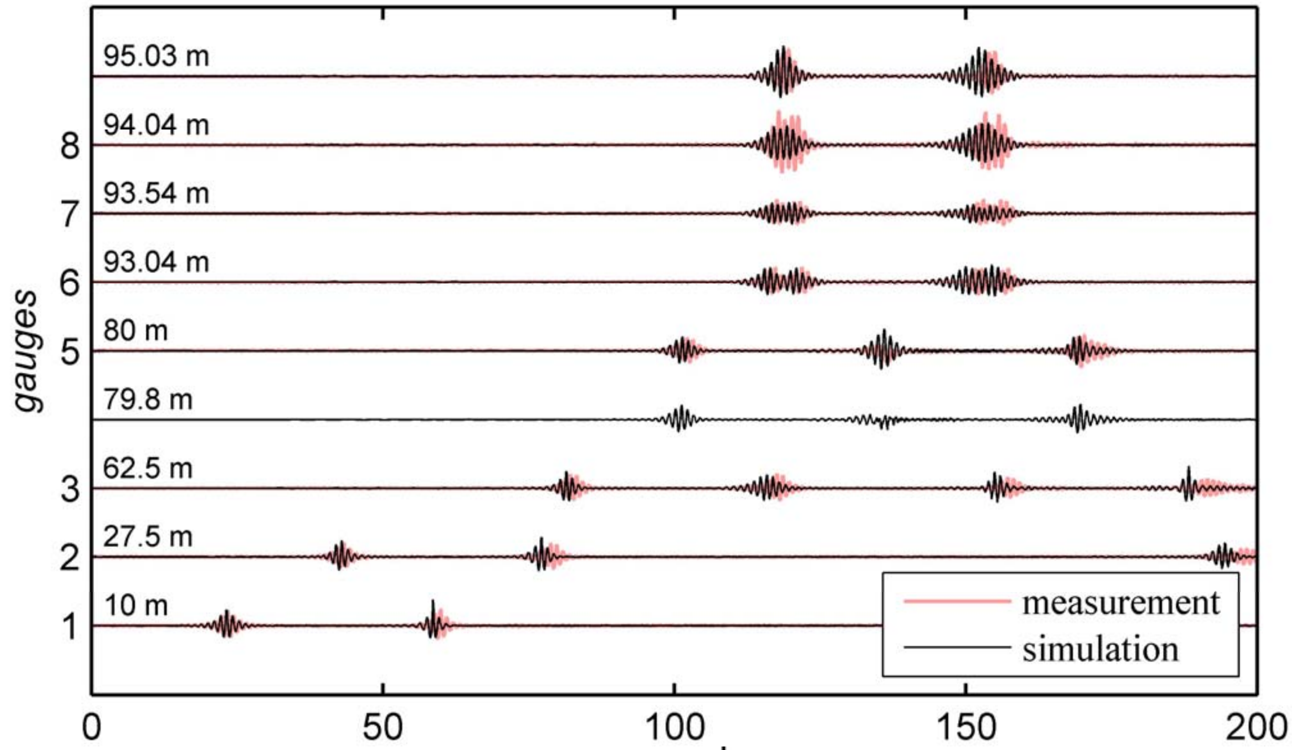


II. Soliton collisions (laboratory)

Overtaking collision (two runs with opposite phases)

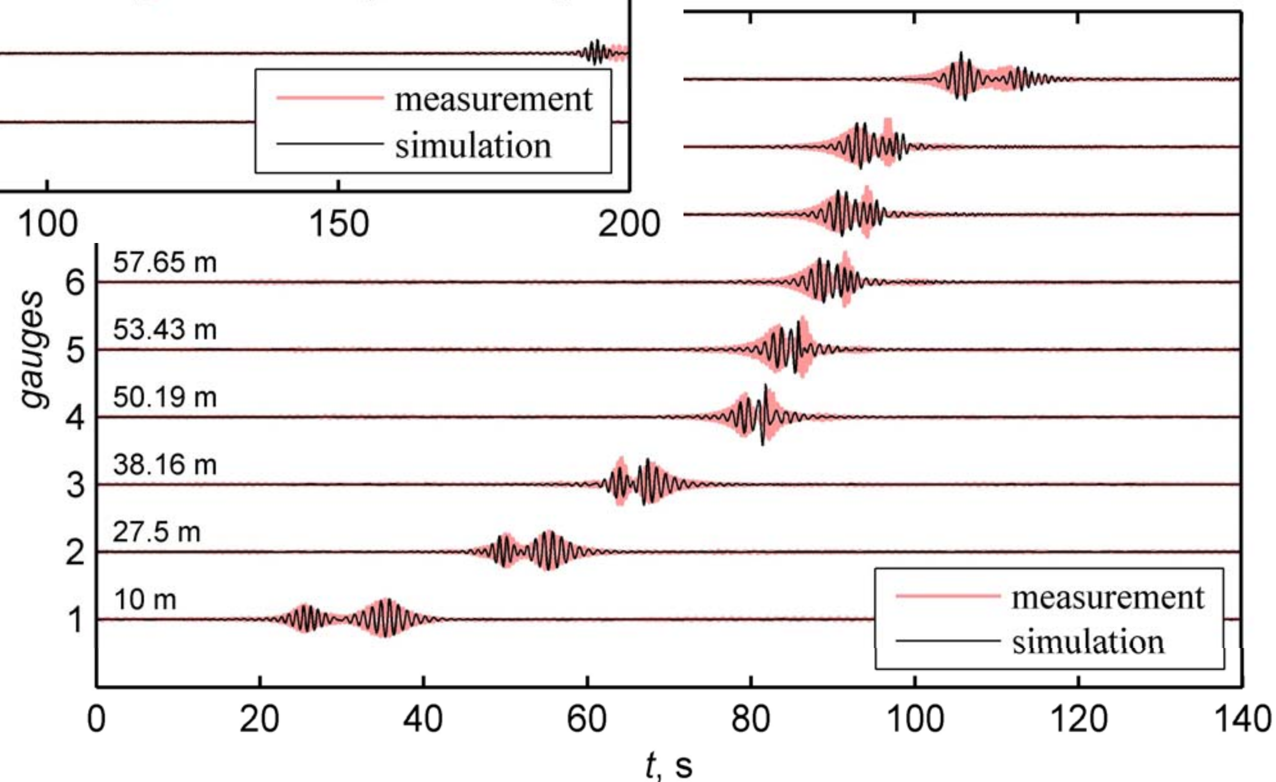


II. Soliton collisions (lab. vs num.)



Possible reasons for the discrepancies:

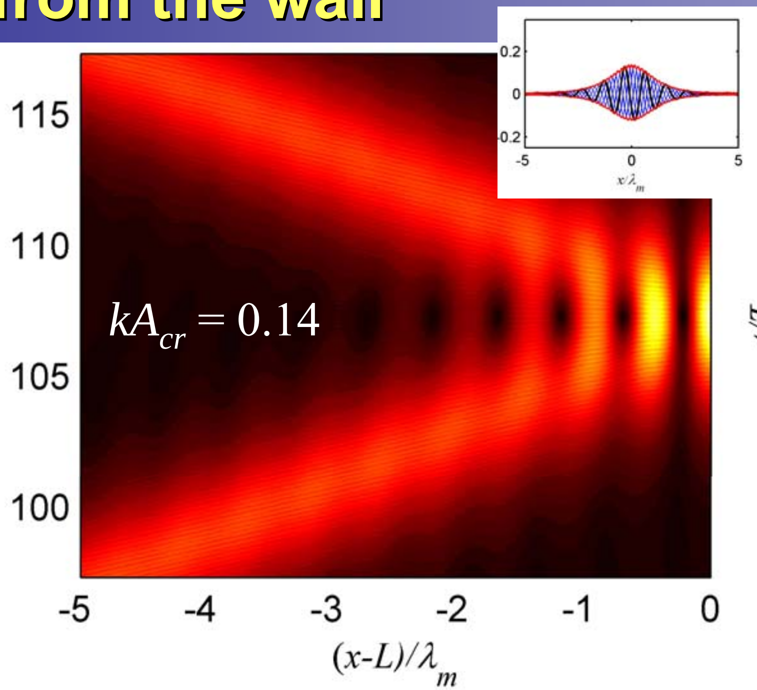
1) transverse effects;
2) sensitivity for the location of the gauges (for fast processes);
3) different wave phases

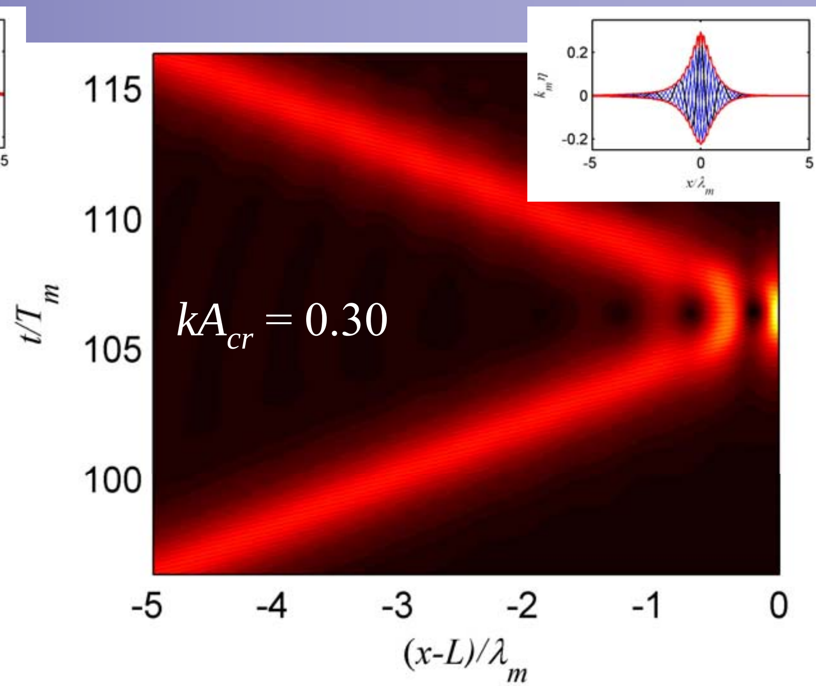


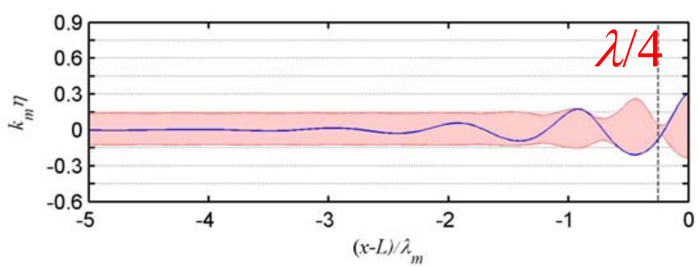
II. Soliton collisions (numerical)

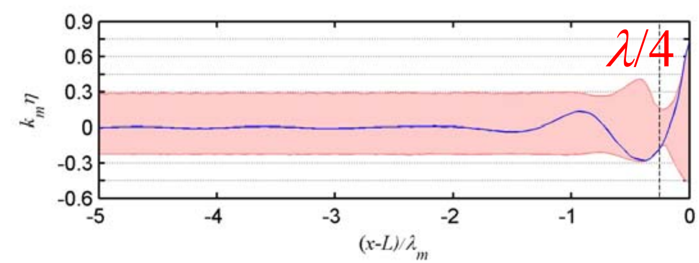
Reflection from the wall

Formation of a standing-wave-like structure of the envelope

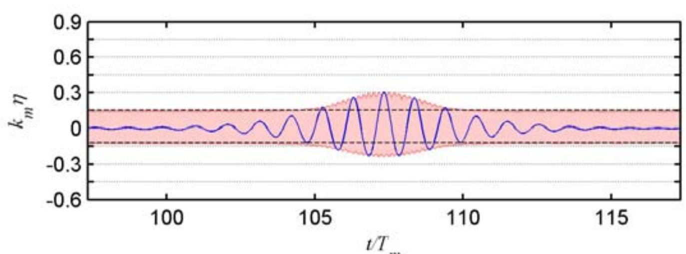


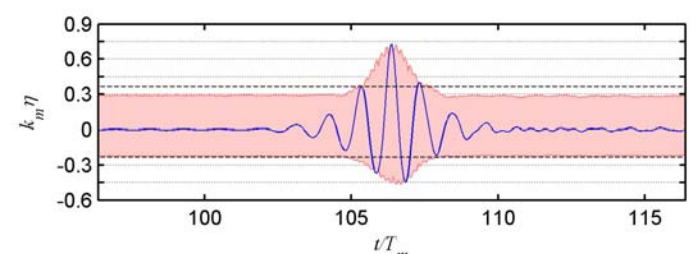






Enhancement at the wall may exceed 2 times





II. Soliton collisions (numerical)

Head-on collisions

snapshots | **time series**

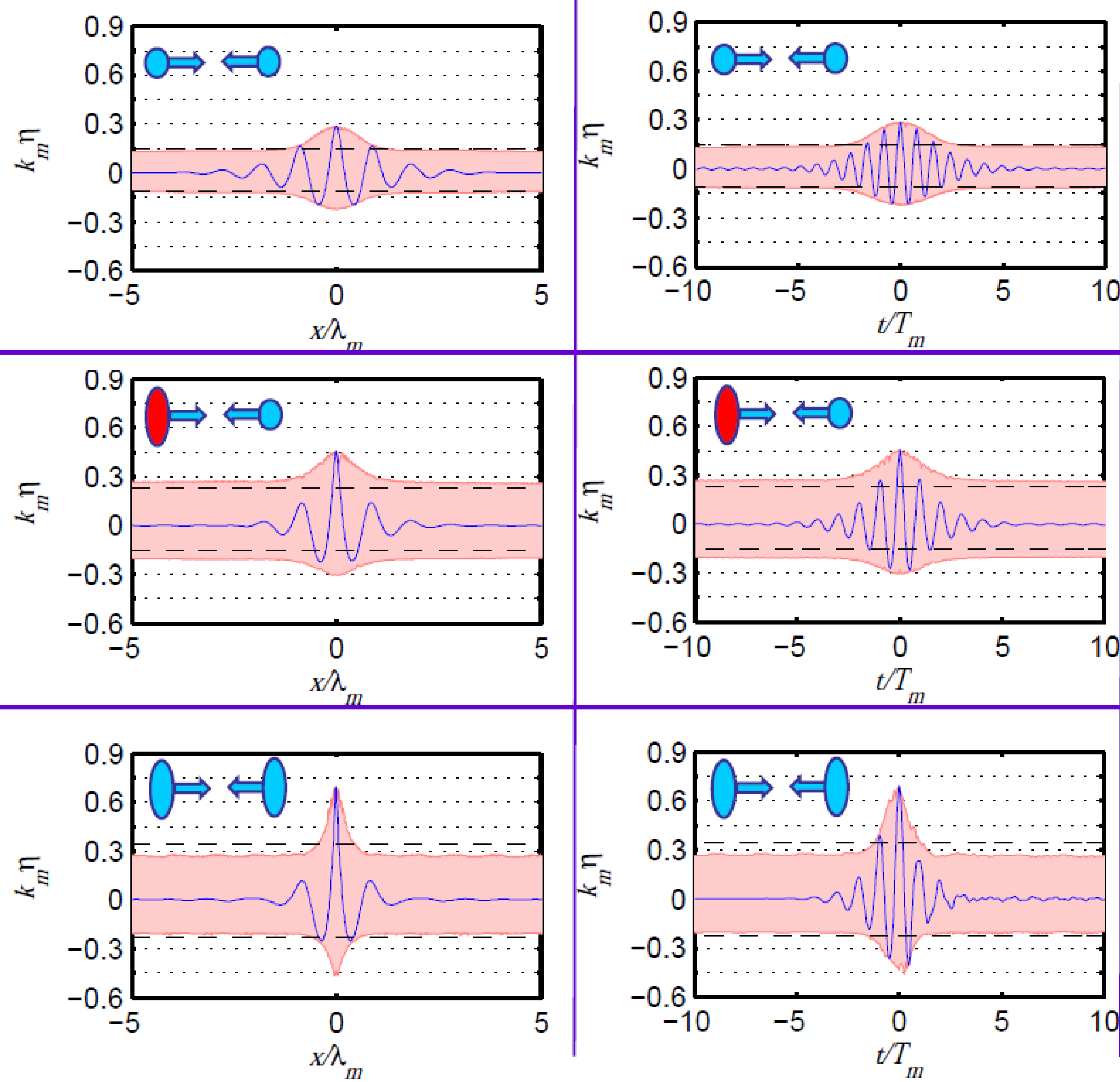
The groups have similar carrier wave lengths

The reflection from a wall is a particular case of a head-on collision. Wave amplification more than 2 times

steepness 0.14 vs 0.14

steepness 0.30 vs 0.14

steepness 0.30 vs 0.30



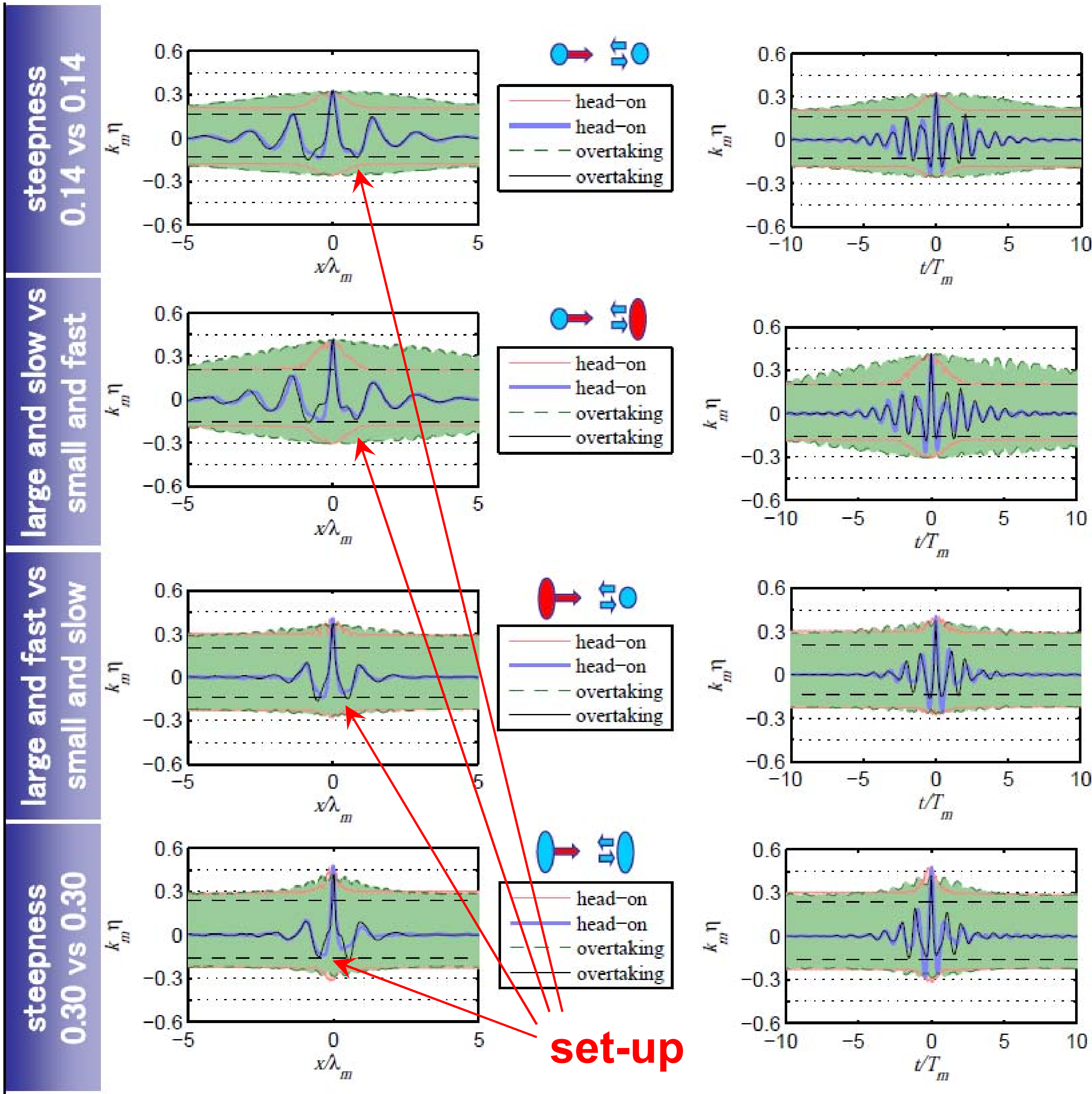
II. Soliton collisions (numerical)

Head-on and overtaking collisions

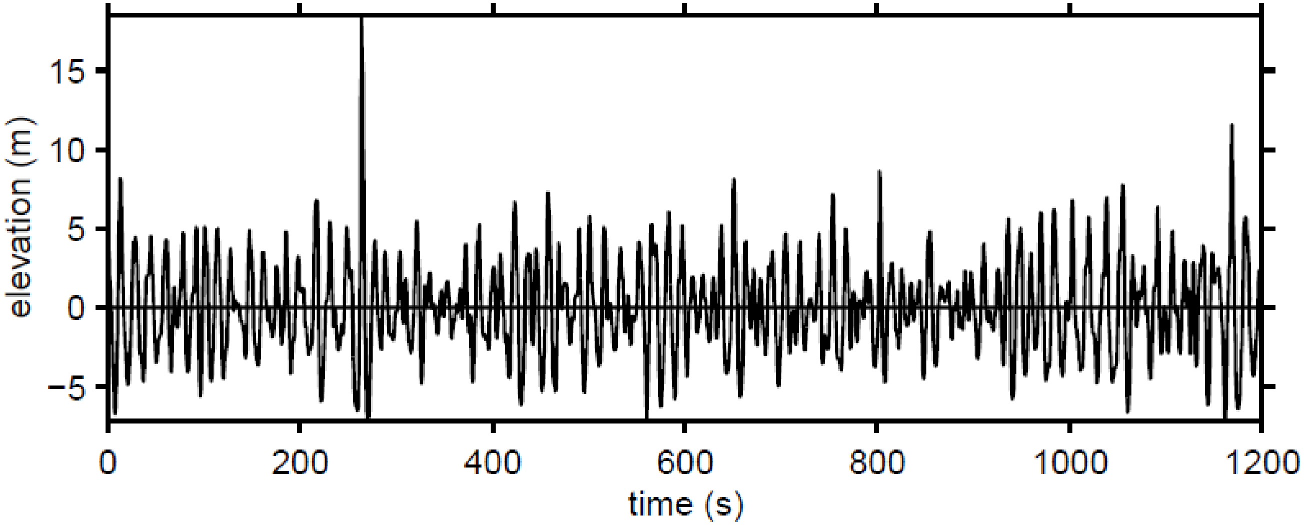
The groups have different carrier wave frequencies ($\Delta\omega/\omega = 0.2$)

When the steepest soliton is the largest one, then the head-on collision results in a larger wave crest than the overtaking collision.

When two interacting solitary groups have different carrier frequencies, the wave with maximum crest experiences an anomalous **set-up**. For unidirectional nonlinear wave groups a set-down is typical.



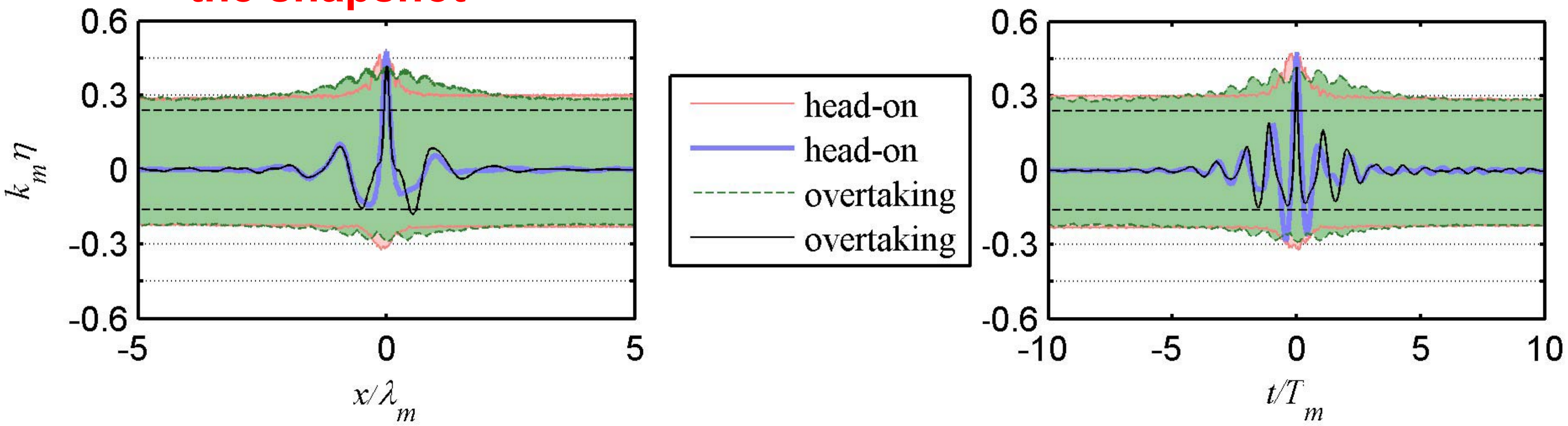
II. Soliton collisions - in the real world (?)



The New Year Wave (Draupner platform, the North Sea) has **a set-up, not a set-down** as could be expected for a NLS nonlinear group. **Adcock et al (2011): crossing seas?**

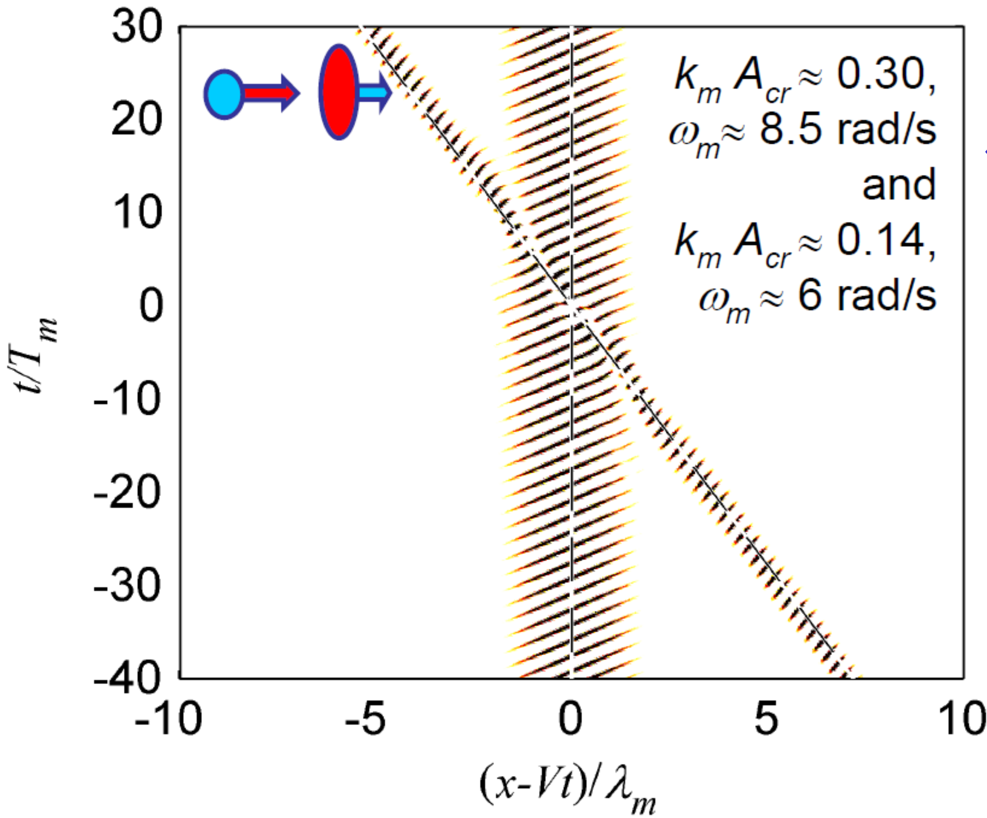
- May be an overtaking collision?

the snapshot

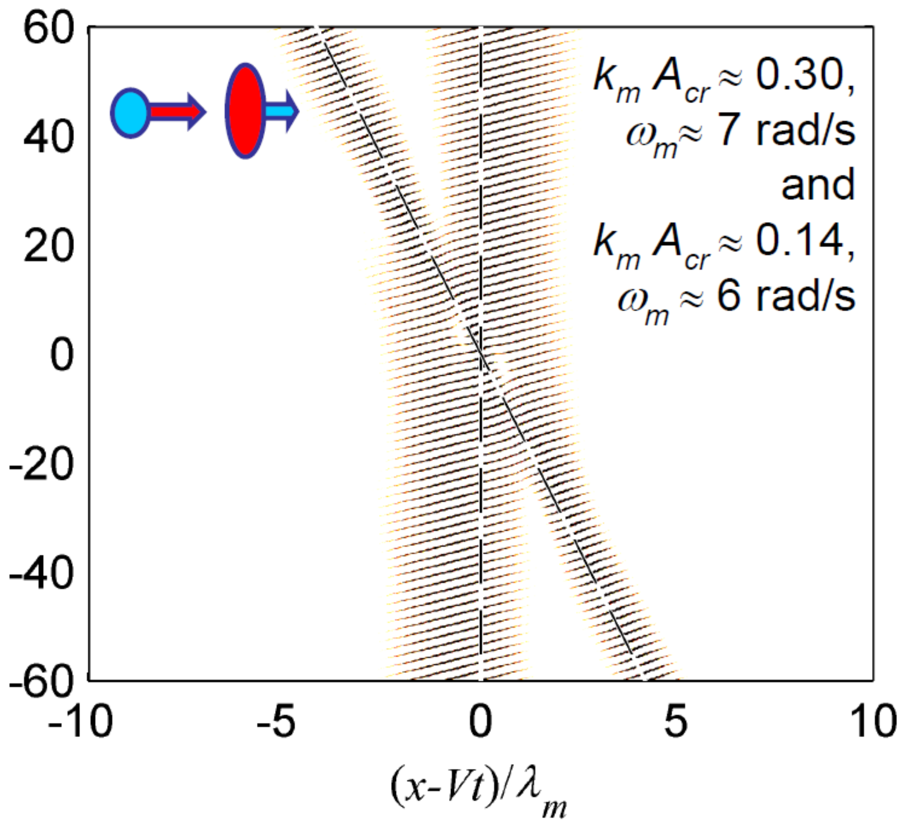


II. Soliton collisions (numerical)

Shifts of the trajectories in overtaking interactions



More distinct carrier frequencies
($\Delta\omega/\omega = 0.2$)



Closer carrier frequencies
($\Delta\omega/\omega = 0.15$)

Conclusions

Such steep wave groups ($kA \approx 0.3$) do not break when collide (!)

The interactions between the intense solitary groups are to a large degree elastic; a shift of envelope solitons' locations is observed

A standing wave structure of the wave envelope appears near the wall

Peculiarities of the interactions between solitary waves groups are revealed. The specific wave shapes (and the associated forces) may make them particularly dangerous.

A. Slunyaev, M. Klein, G.F. Clauss, Phys. Fluids (2017, in press)