Decoherence in a Cooper pair Shuttle

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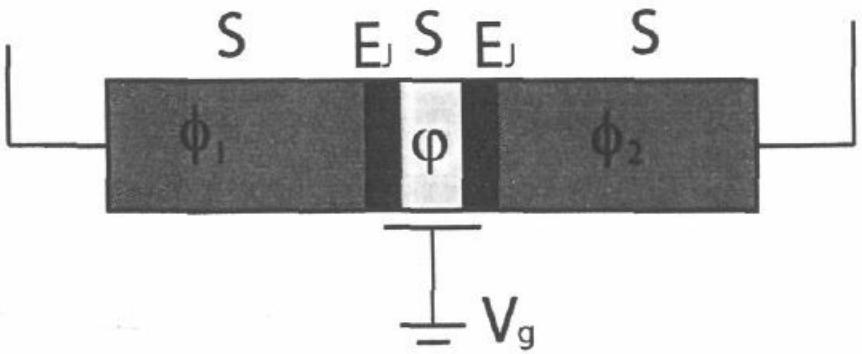
Outline

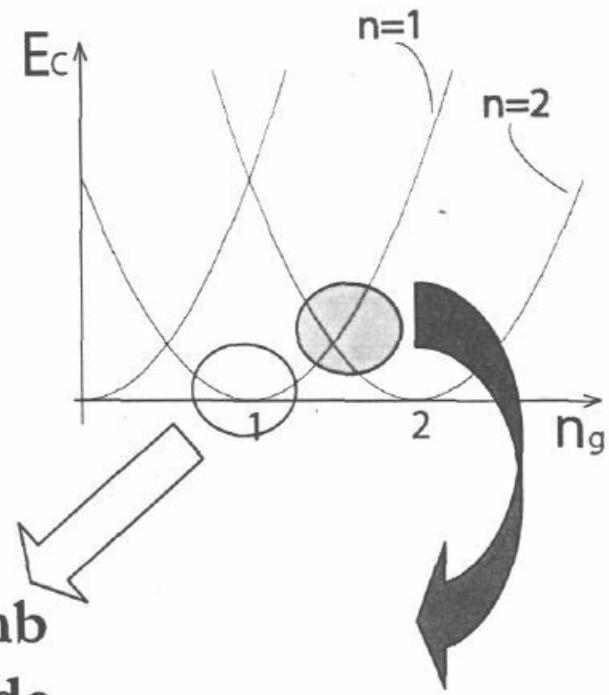
- **Josephson vs Coulomb Blockade**
- **Nanomechanical systems and shuttles**
- **Cooper pair shuttle**
- **Decoherence**

Josephson current in a S-SET

$$H_C = E_C\left(\hat{n} - n_g\right)^2$$

$$I = I_J \sin\left(\phi_1 - \phi_2\right)$$





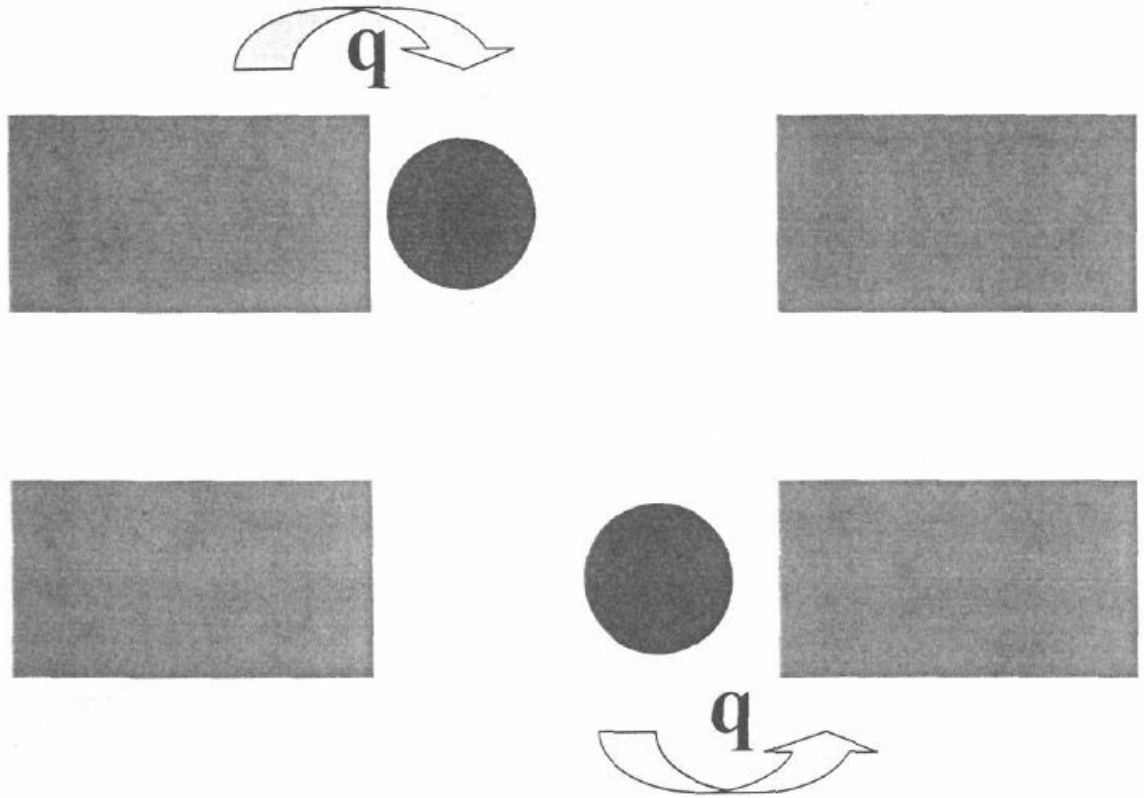
The critical current depends on the charging energy of the island

Coulomb blockade

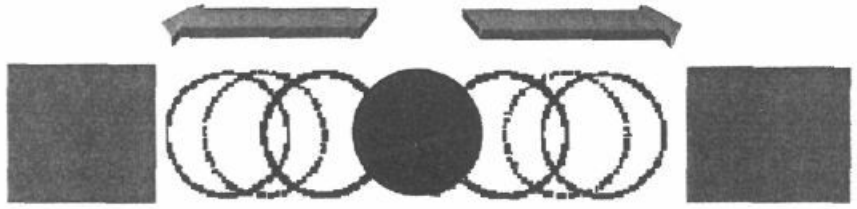
Degeneracy

Single Electron effects in the presence of mechanical vibrations

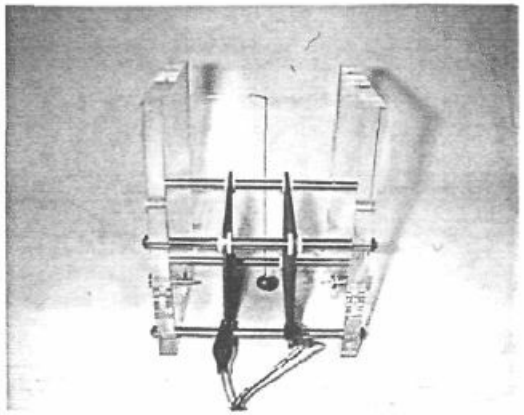
Gorelik et al 1998

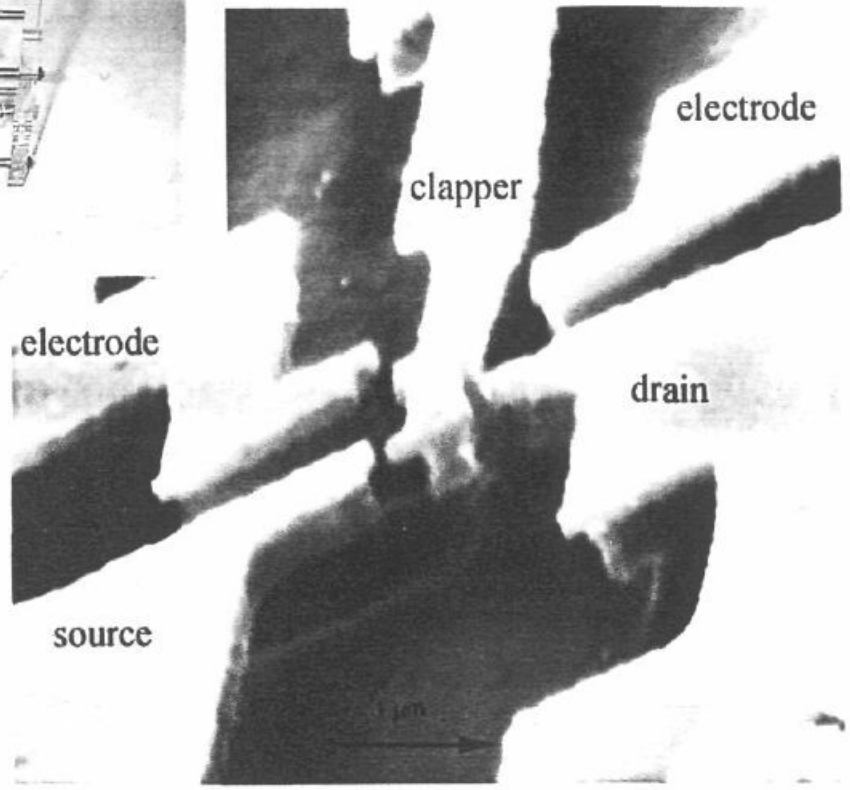


SHUTTLE EFFECT



Experiments





- Tuominen et al 1999
- Park et al 2000
- Erbe et al 2001

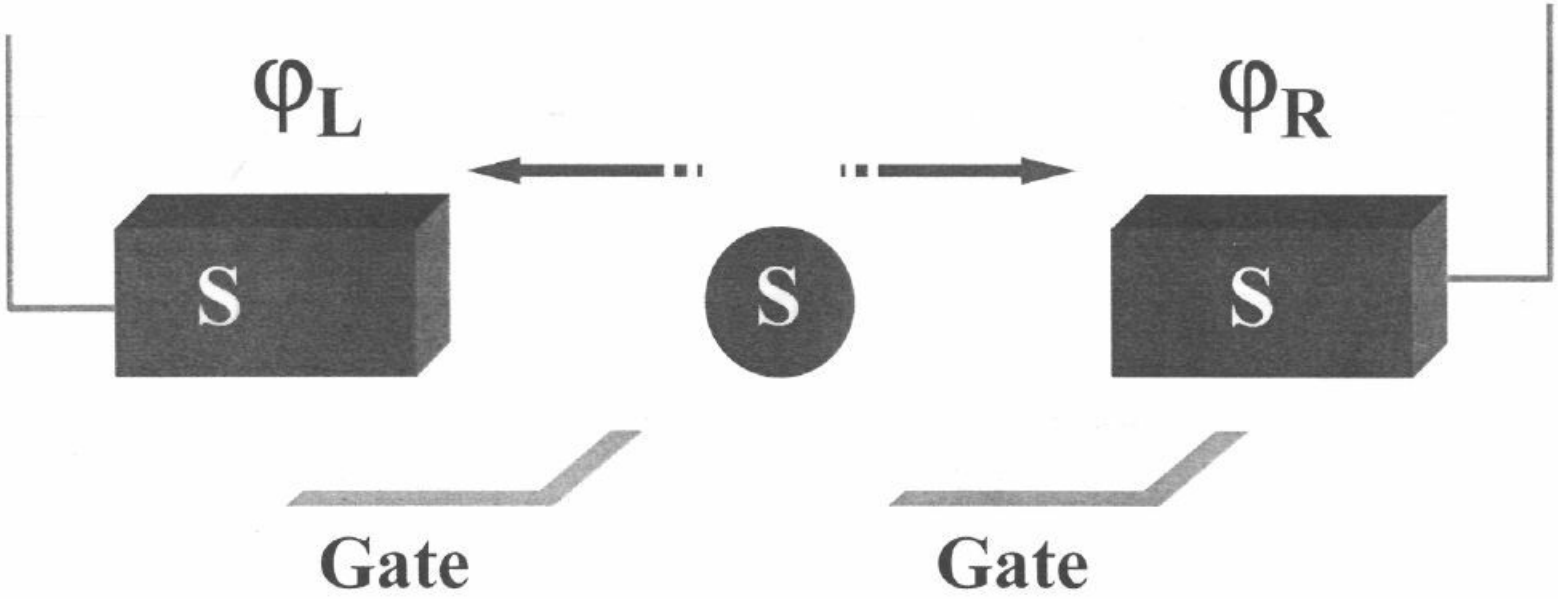
Superconducting Electrodes

- Shuttle effect with Cooper pairs?

Gorelik et al 2001

- Is it a coherent effect?

- What is the role of decoherence?



A single Cooper-pair box, by periodically moving between two superconducting leads, is able to keep phase coherence of the two distant electrodes

The Model

$$H = E_C (n - n_g(t))^2 - \sum_{\alpha=L,R} E_J^{\alpha}(t) \cos(\varphi - \varphi_\alpha)$$

The external leads have well defined phase

- **The system is in regime of strong Coulomb blockade ($E_J << E_C$). The Hilbert space of the shuttle is two-dimensional, spanned by two charge states differing by one Cooper-pair.**

$t_A \le t \le t_B$

$E_J^L \ne 0, E_J^R = 0, n_g = \frac{1}{2}$

Free evolution

$t_B \le t \le t_C$

$E_J^L = 0, E_J^R = 0, n_g = 0$

$t_C \le t \le t_D$

$E_J^L = 0, E_J^R \ne 0, n_g = \frac{1}{2}$

$t_D \le t \le t_A + T$

$E_J^L = 0, E_J^R = 0, n_g = 0$

"Josephson hybrid"

$$|\psi\rangle = \alpha|0\rangle + \beta e^{i\varphi_{L/R}}|1\rangle$$

Accumulated phases

- Phase difference

$$\varphi = \varphi_R - \varphi_L$$

- Dynamical phases

$$\vartheta = \frac{1}{2}\int_A^B E_J(t)dt$$

$$\chi = \frac{1}{2}\int_B^C E_C(t)dt$$

Decoherence

- Gate fluctuations
- Background charges
- Quasi particle tunneling
- ...

Important in establishing a stationary state

Strongly affects the Josephson current

Coupling to the environment

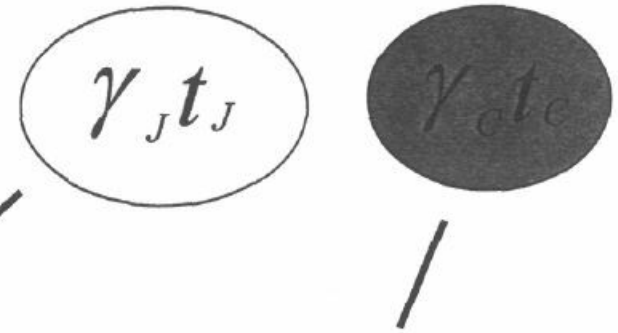
$$H_{\text{coupling}} = \sum_i \lambda_i \left(b_i + b_i^+ \right)$$

Treated in Born-Markov approximation

Master equation for the density matrix ρ in the space generated by $|0\rangle, |1\rangle$ states

Steady state : fixed point for the map $\rho\,(\mathrm{nT}) \rightarrow \rho\,(\mathrm{nT} + \mathrm{T})$

The effect of the environment are taken into account by



Contact region

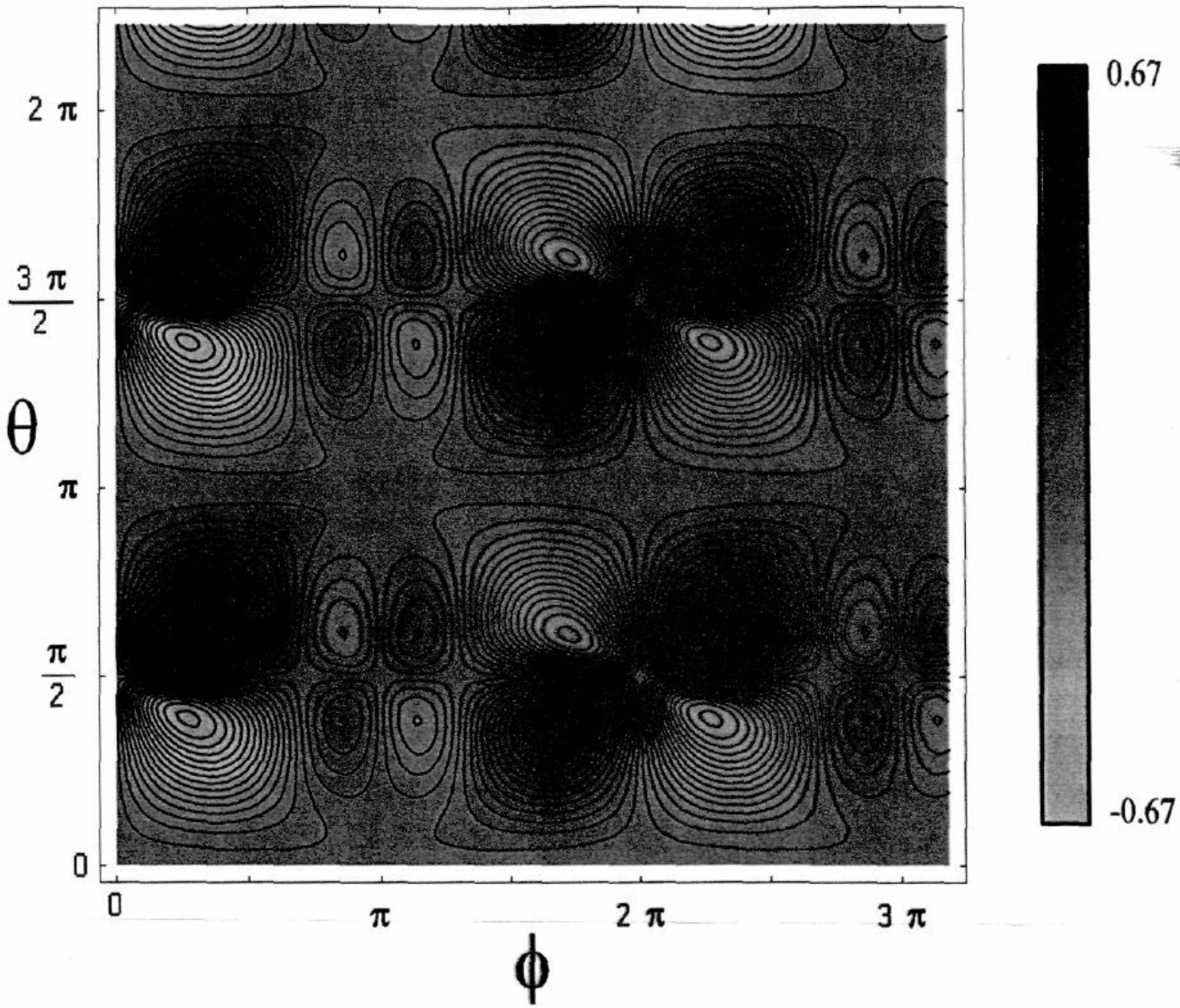
Free evolution region

Modification of the critical current:

$$I = I\left(\phi, \vartheta, \chi, \gamma_J t_J, \gamma_C t_C\right)$$

Average current over a period:

$$I = \frac{1}{T} \int_{\text{period}} \left\langle \hat{I}(t) \right\rangle dt$$


2 π
3 π / 2
θ
π
π / 2
0
0
π
2 π
3 π
ϕ
0.67
-0.67

Weak damping $\gamma_J t_J << \gamma_C t_C << 1$

$$I \approx \frac{2e}{T}\frac{\gamma_J t_J}{\gamma_C t_C}\frac{(\cos\varphi + \cos 2\chi)\tan\vartheta}{1+\cos\varphi\cos 2\chi}\sin\varphi$$

Current is suppressed if dephasing in the contact region tends to zero

Strong damping

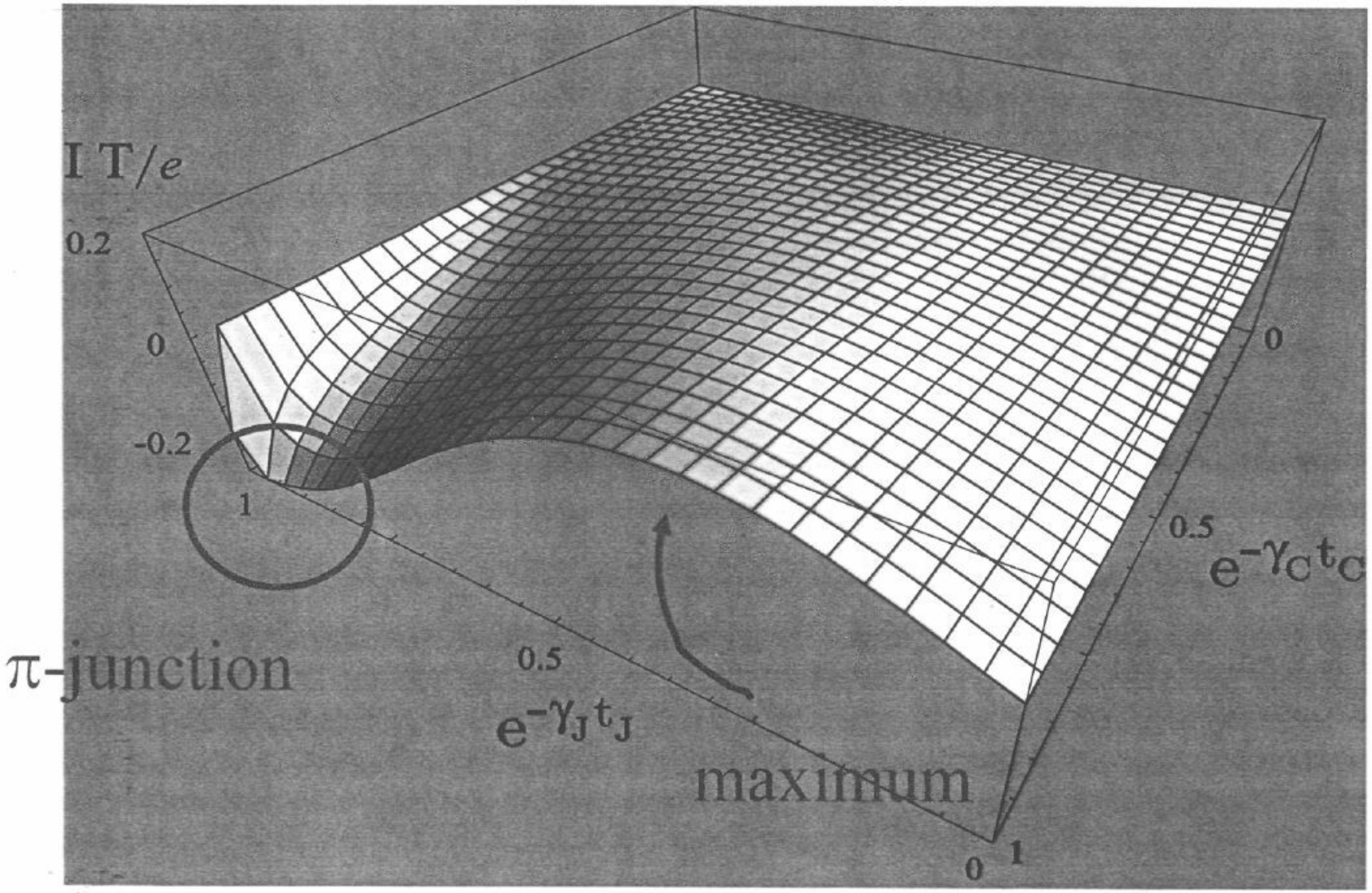
$$\gamma_J t_J >> 1,\ \gamma_C t_C >> 1$$

$$I = \frac{2e}{T}\tanh\left(2\beta E_J\right)e^{-(\gamma_J t_J + \gamma_J t_J)}\cos(2\chi)\sin(2\vartheta)\sin(\phi)$$

If the dephaing is strong enough the coherence is lost before the period is completed: the current is exponentially suppressed

Change in sign

Dependence on the dephasing rates



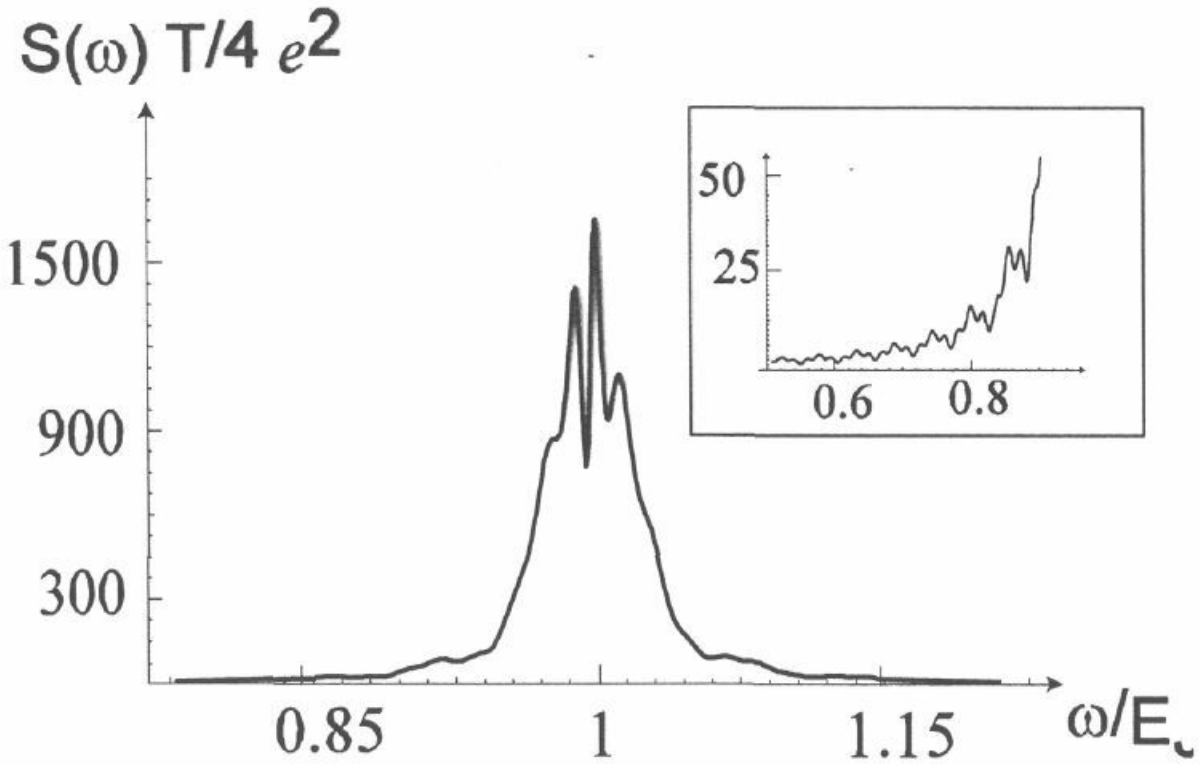
Zero-frequency noise

Strong damping $$S(0) = \frac{4e^2}{T}\left\{\frac{1}{2} - e^{-\gamma_J t_J}\cos\theta + e^{-2\gamma_J t_J}\,\mathrm{f}\left(\vartheta,\chi,\phi,\ldots\right)\right\}$$

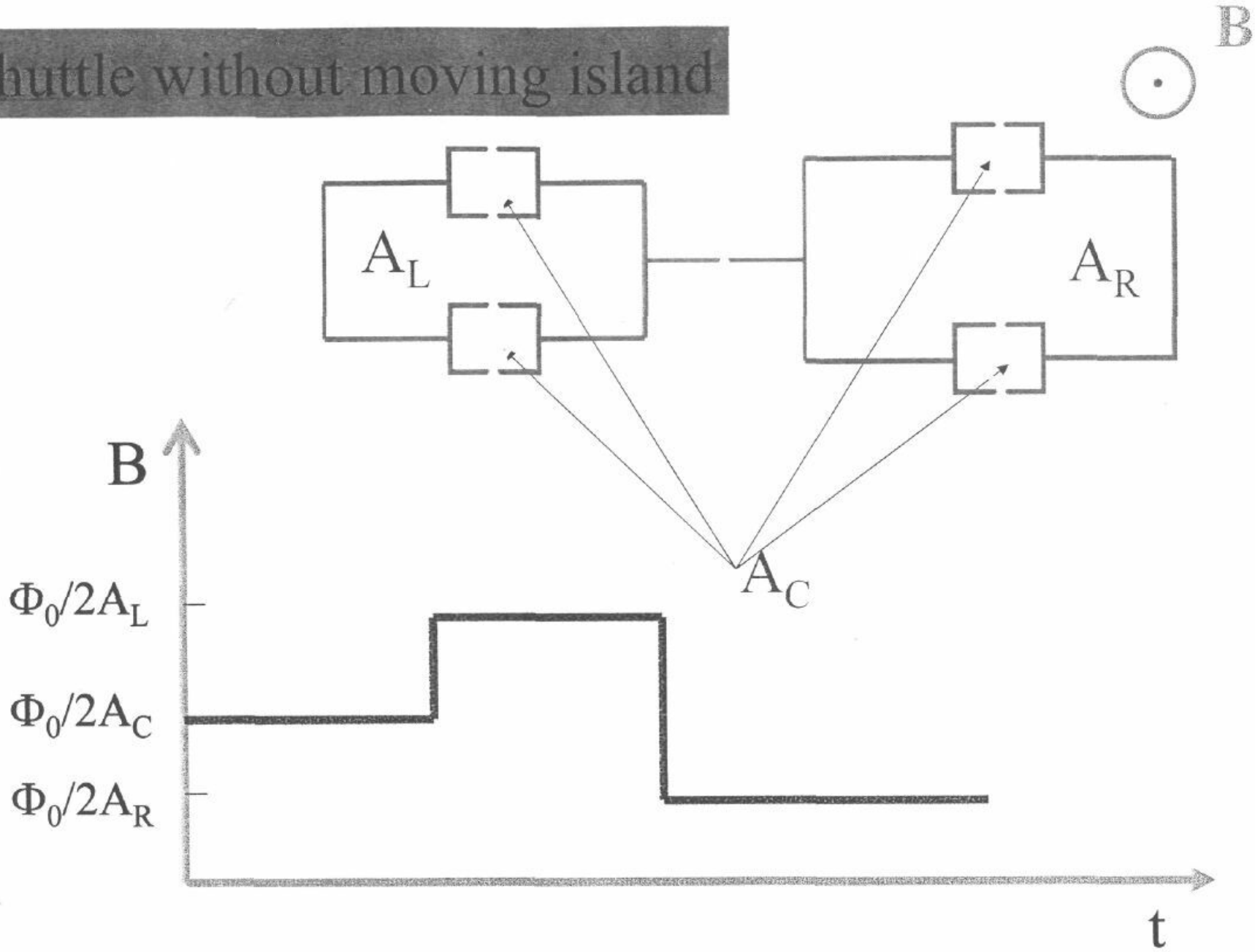
Weak damping $$S(0) = \frac{4e^2}{T}\frac{1}{\gamma_C t_C}\frac{\tan^2\vartheta\,\sin^2\phi}{2\left(1+\cos\phi\cos(2\chi)\right)}$$

Current noise

$$S(\omega)=\frac{1}{T}\int_0^T dt\int_{-\infty}^{+\infty} d\tau\left\{\frac{1}{2}\left\langle\hat{I}(t+\tau)\hat{I}(t)\right\rangle-\left\langle\hat{I}(t+\tau)\right\rangle\left\langle\hat{I}(t)\right\rangle\right\}e^{-i\omega\tau}$$



Shuttle without moving island



Conclusions

- The Cooper pair shuttle is able to mantain and create phase coherence between two "distant" superconductors.
- Fluctuations of gate voltage can either enhance or suppress the supercurrent.
- The Cooper pair shuttle can [redacted] behave as a π-junction behavior.
- The current noise displays a peak at Josephson energy.