

Measurement of Non-Gaussian Shot Noise – Influence of the environment

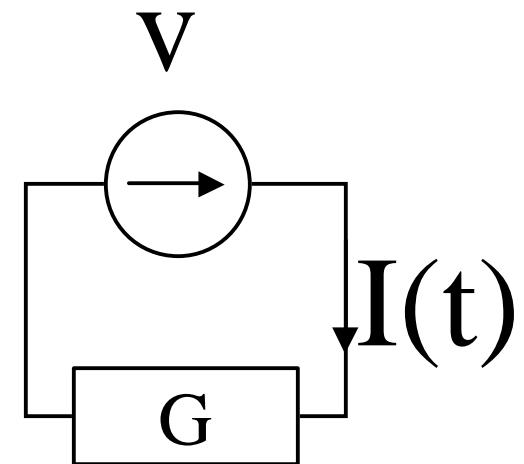
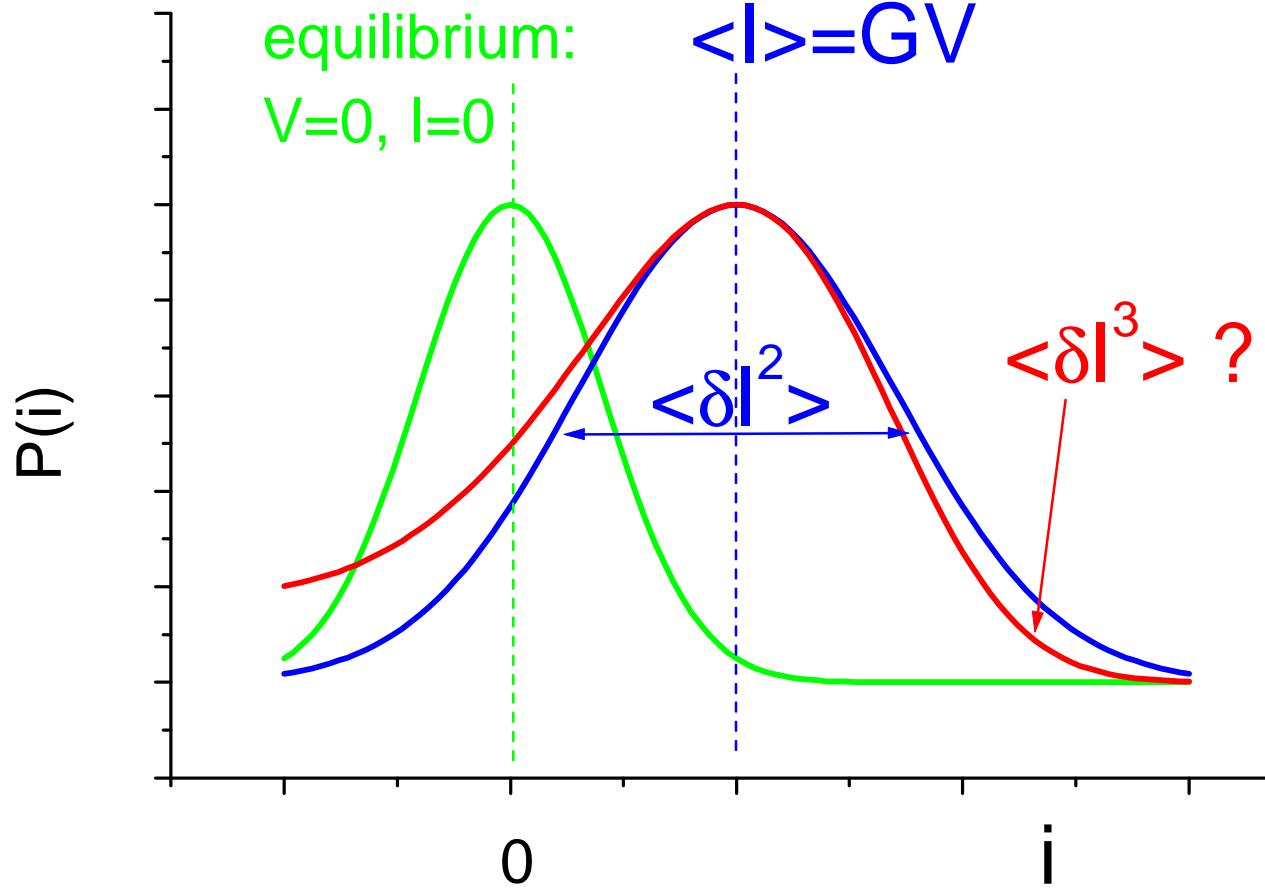
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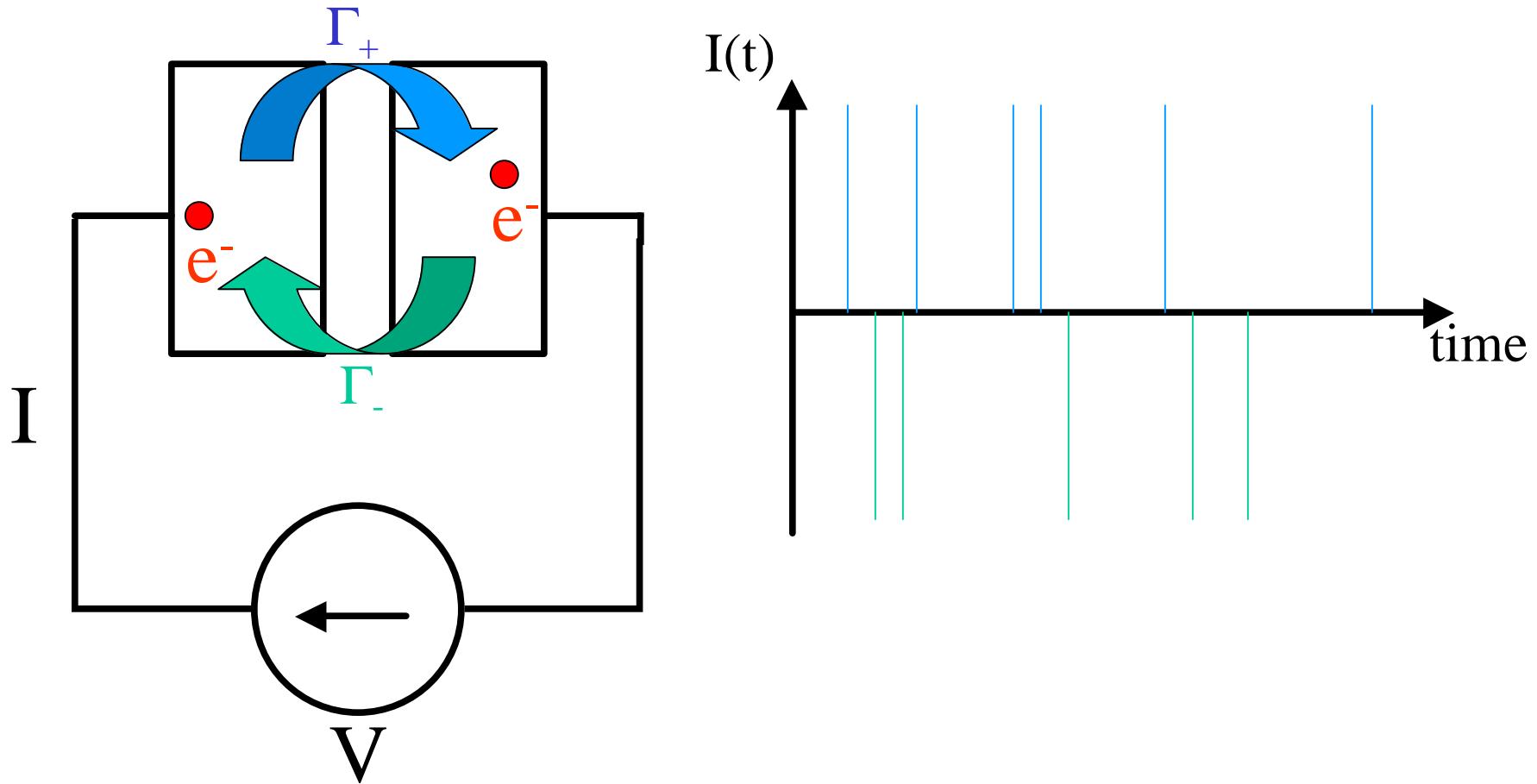
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Statistics of the current



$$I(t) = \langle I \rangle + dI(t)$$

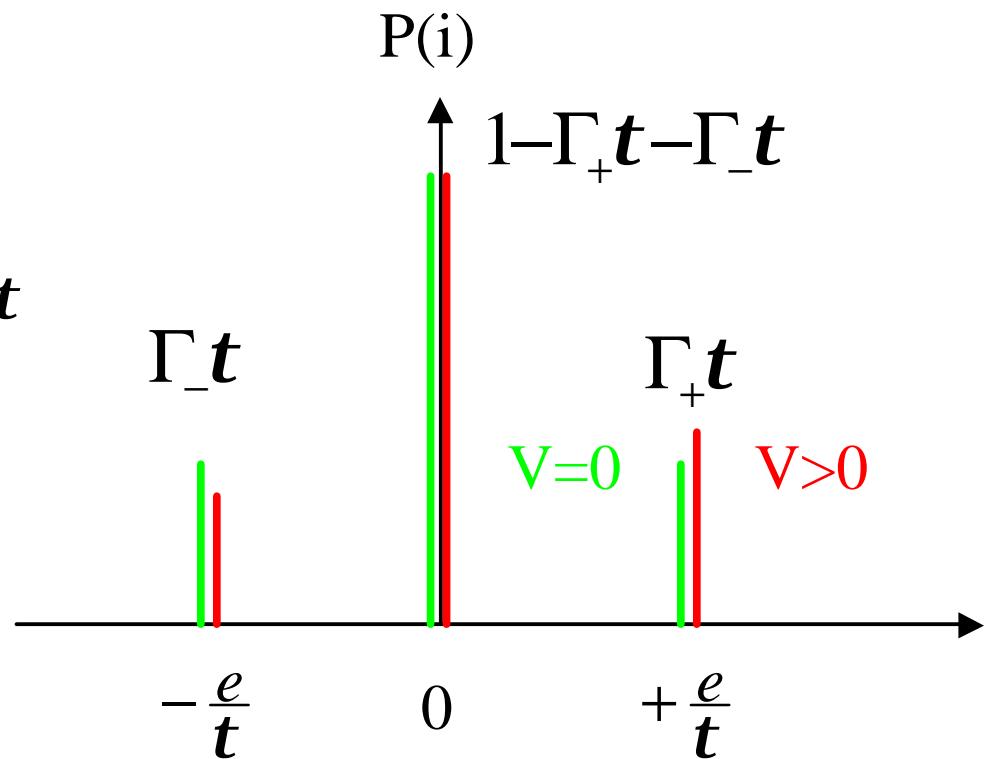
Current in a tunnel junction (single channel)



Statistics of the Current in a tunnel junction (single channel)

Each time τ :

$$P(I) = \begin{cases} P(+1e) = \Gamma_+ t \\ P(-1e) = \Gamma_- t \\ P(0e) = 1 - \Gamma_+ t - \Gamma_- t \end{cases}$$



$$\langle I \rangle = \left(+\frac{e}{t} \right) \Gamma_+ t + \left(-\frac{e}{t} \right) \Gamma_- t = e(\Gamma_+ - \Gamma_-) = GV$$

S_3 for a tunnel junction

$$\langle I^3 \rangle = \left(+\frac{e}{t} \right)^3 \Gamma_+ t + \left(-\frac{e}{t} \right)^3 \Gamma_- t = \frac{e^3}{t^3} (\Gamma_+ t - \Gamma_- t) = \frac{e^2}{t^2} \langle I \rangle$$

Spectral density: $S_3 = e^2 I [A^3/Hz^2]$

Full expression (single channel):

$$S_3 = \frac{e^3}{h} t (1-t) k_B T (6tf(U) + (1-2t)U)$$

$$= e^2 (1-t)(1-2t) I + 6tGk_B T \frac{dS_2}{dI}$$

$$\text{with } U = \frac{eV}{k_B T}$$

Transmission $t \sim 10^{-5}$

Lesovic, Levitov

Signal to Noise ratio

Ingredients: integration time τ , bandwidth B, amplifier's noise S_A

Average current: $x_{\langle I \rangle} = \sqrt{Bt} \frac{I}{(S_A B)^{\frac{1}{2}}}$ independent of B

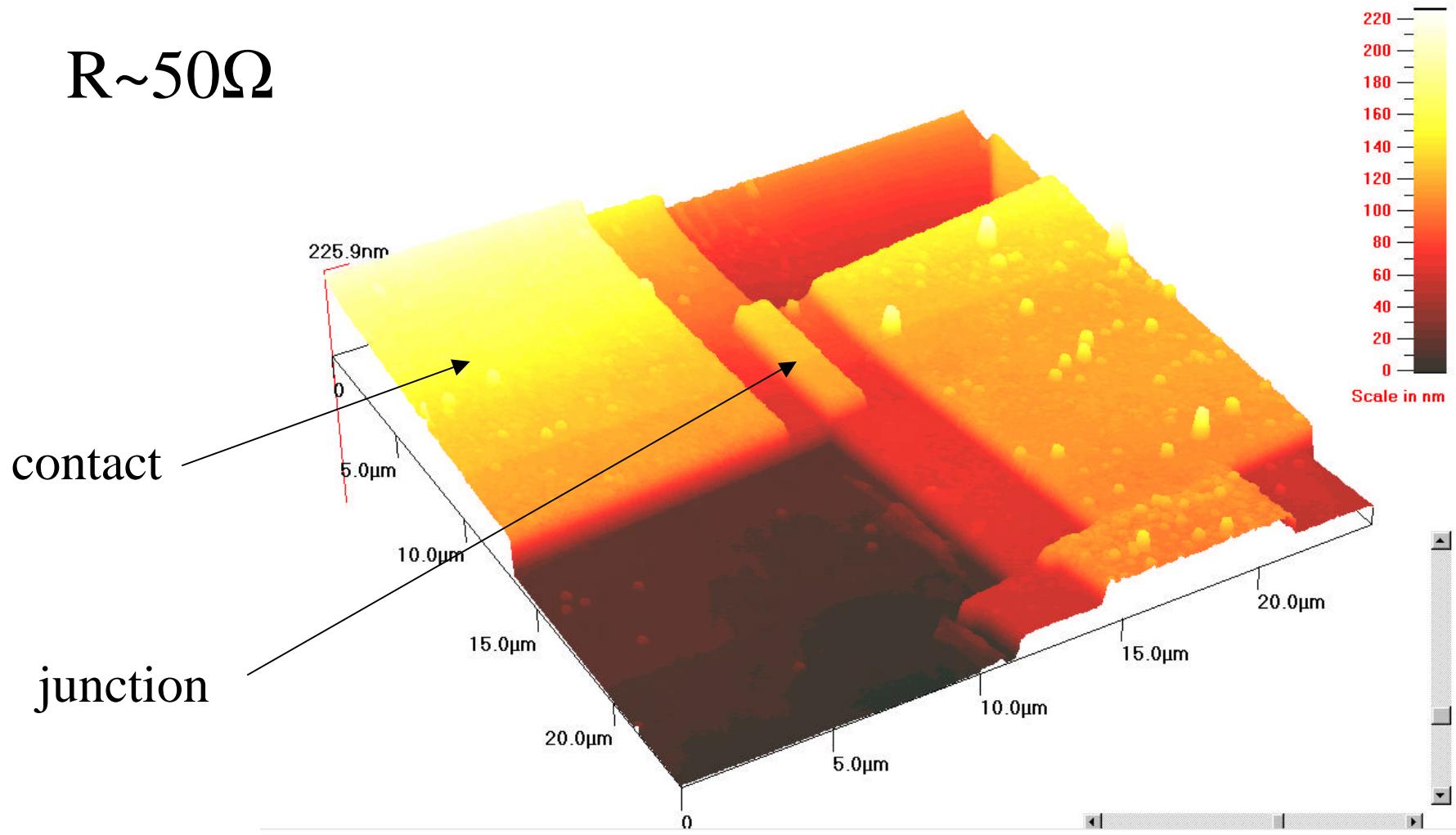
Noise S_2 : $x_{\langle dI^2 \rangle} \propto \sqrt{Bt} \frac{eIB}{(S_A B)} = x_{\langle I \rangle} \left(\frac{eB}{\sqrt{S_A B}} \right)$

Noise S_3 : $x_{\langle dI^3 \rangle} \propto \sqrt{Bt} \frac{e^2 IB^2}{(S_A B)^{\frac{3}{2}}} = x_{\langle I \rangle} \left(\frac{eB}{\sqrt{S_A B}} \right)^2$

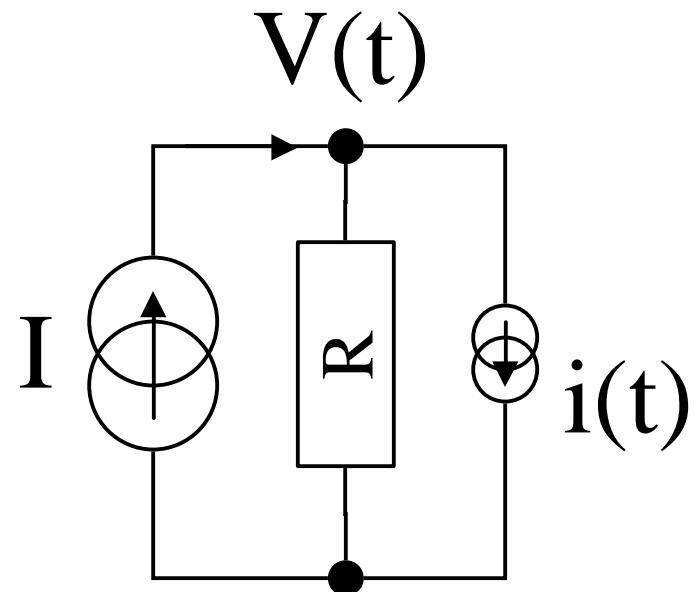
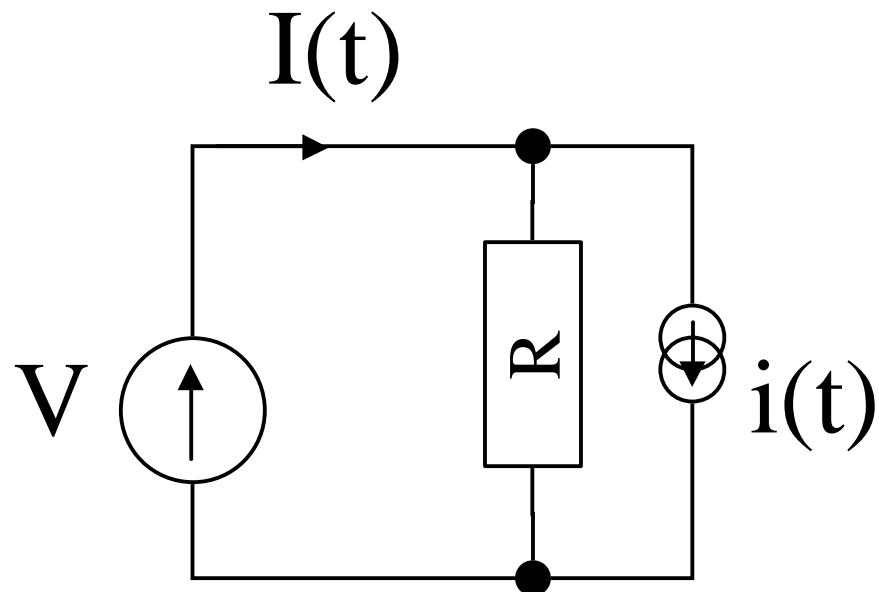
$$\left(\frac{eB}{\sqrt{S_A B}} \right) \approx 10^{-3} - 10^{-4}$$

The sample : Al/Al₂O₃/Al

R~50Ω



Voltage bias vs. Current bias



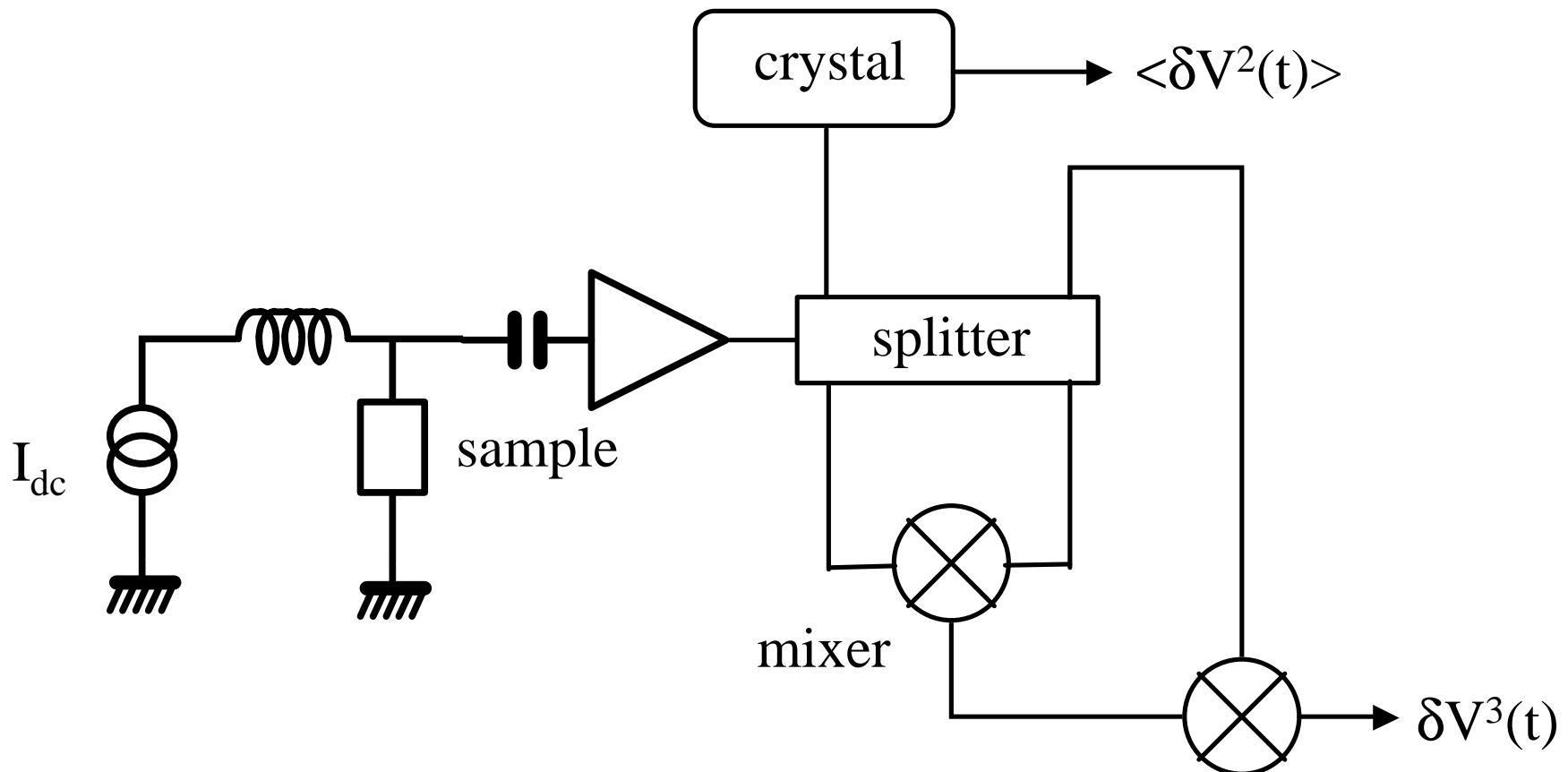
$$dI(t) = i(t)$$

$$dV(t) = -Ri(t)$$

$$V = RI$$

$$\langle dV \rangle^2 = R^2 \langle dI \rangle^2$$

Measurement: method

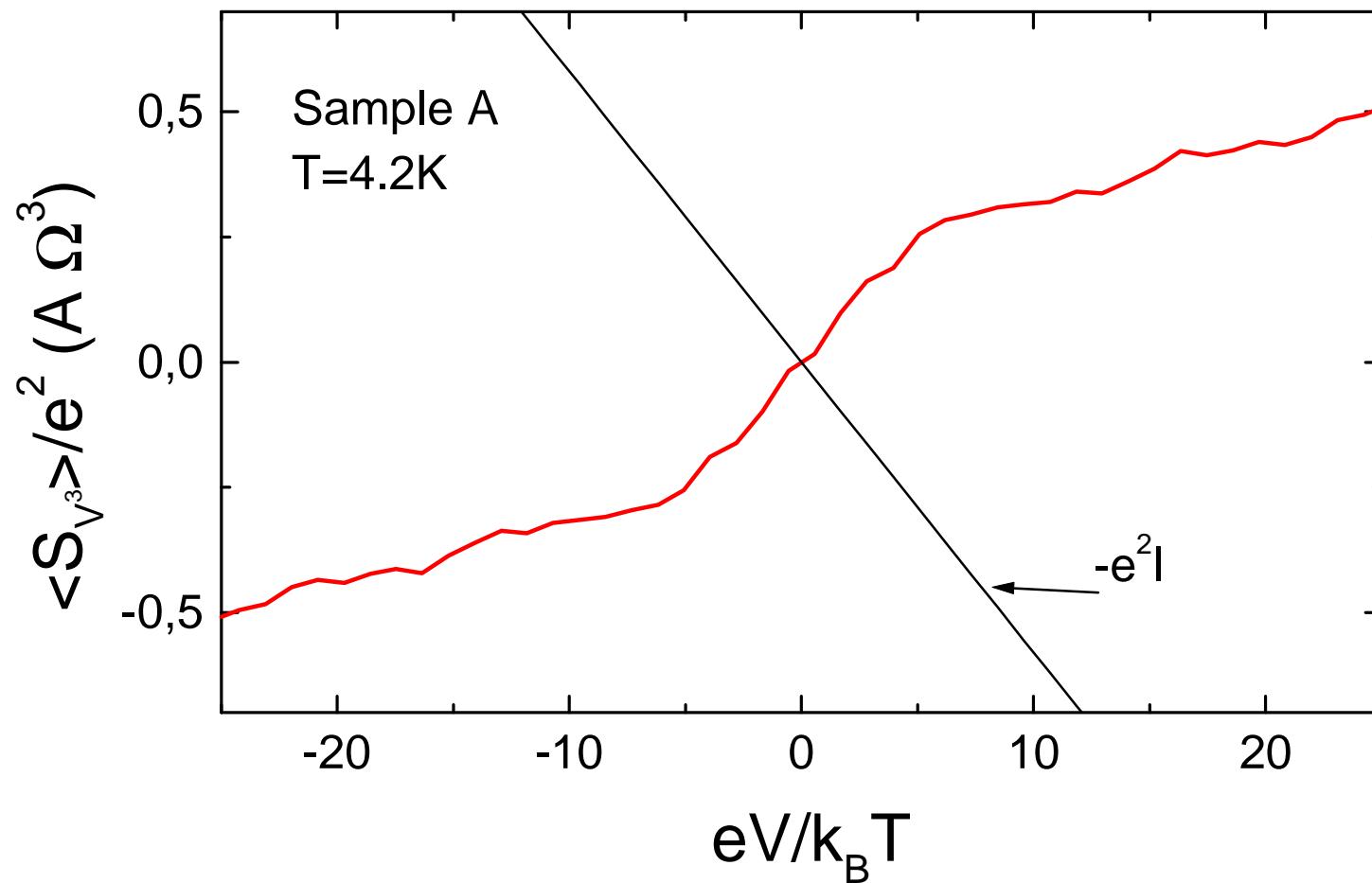


Bandwidth: 10-1000 MHz

Experimental challenges

- Imperfections of the mixers
- Non linearities of the amplifiers
- Oscillations, external noise sources
- Asymmetry of S_2 of the sample
- Time delay between different branches
- Effect of sample's noise on the bias point of the mixers
- Calibration

Result (sample A, T=4.2K)



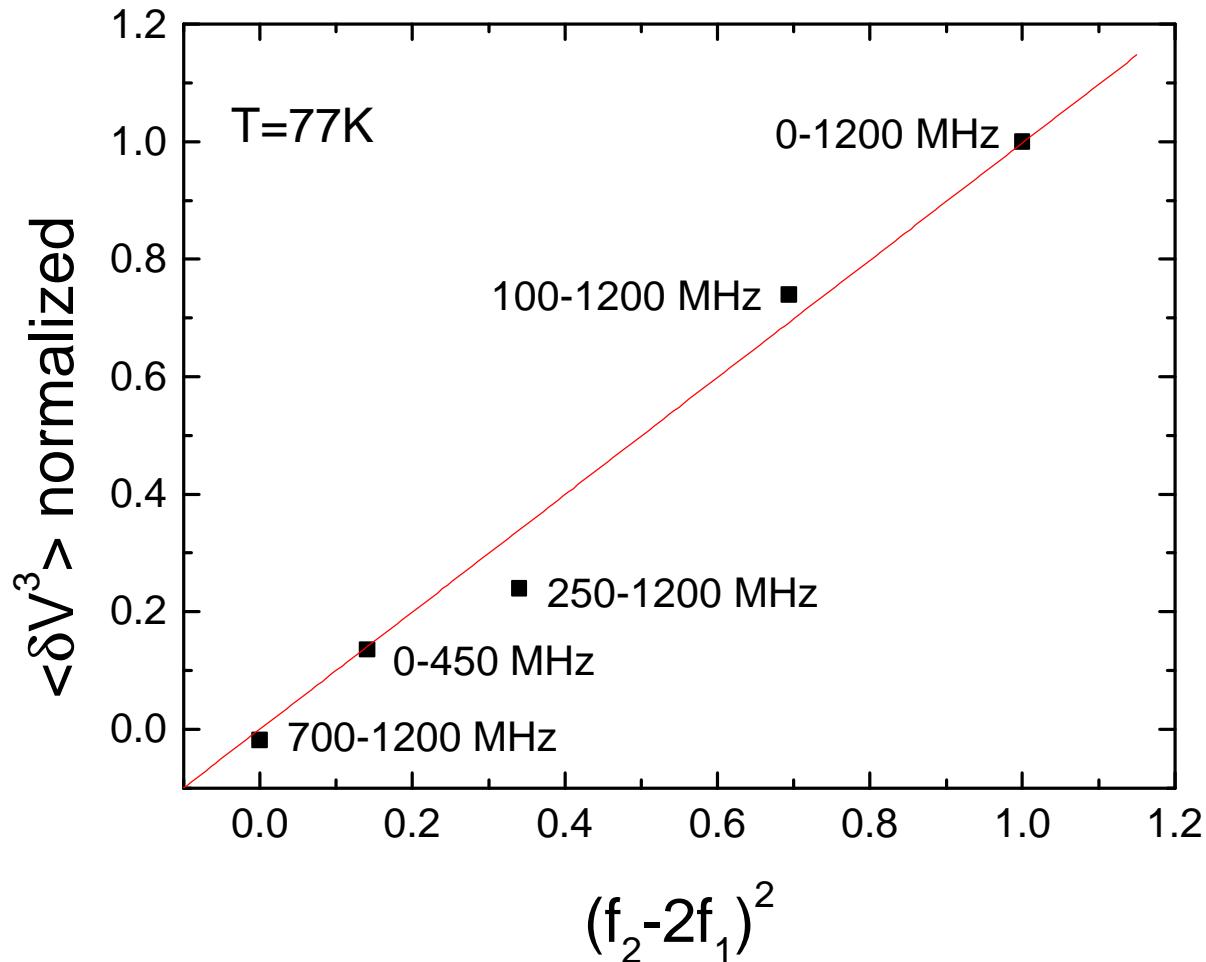
Importance of the bandwidth

$$\begin{aligned} \langle \mathbf{d}I(t)^2 \rangle &= \iint d\mathbf{w} d\mathbf{w}' \overbrace{I(\mathbf{w})I(\mathbf{w}') \mathbf{d}(\mathbf{w} + \mathbf{w}')}^{S_2(\mathbf{w})} = \int_{\mathbf{w}_1}^{\mathbf{w}_2} d\mathbf{w} |I(\mathbf{w})|^2 \\ &\propto (\mathbf{w}_2 - \mathbf{w}_1) S_2(0) \end{aligned}$$

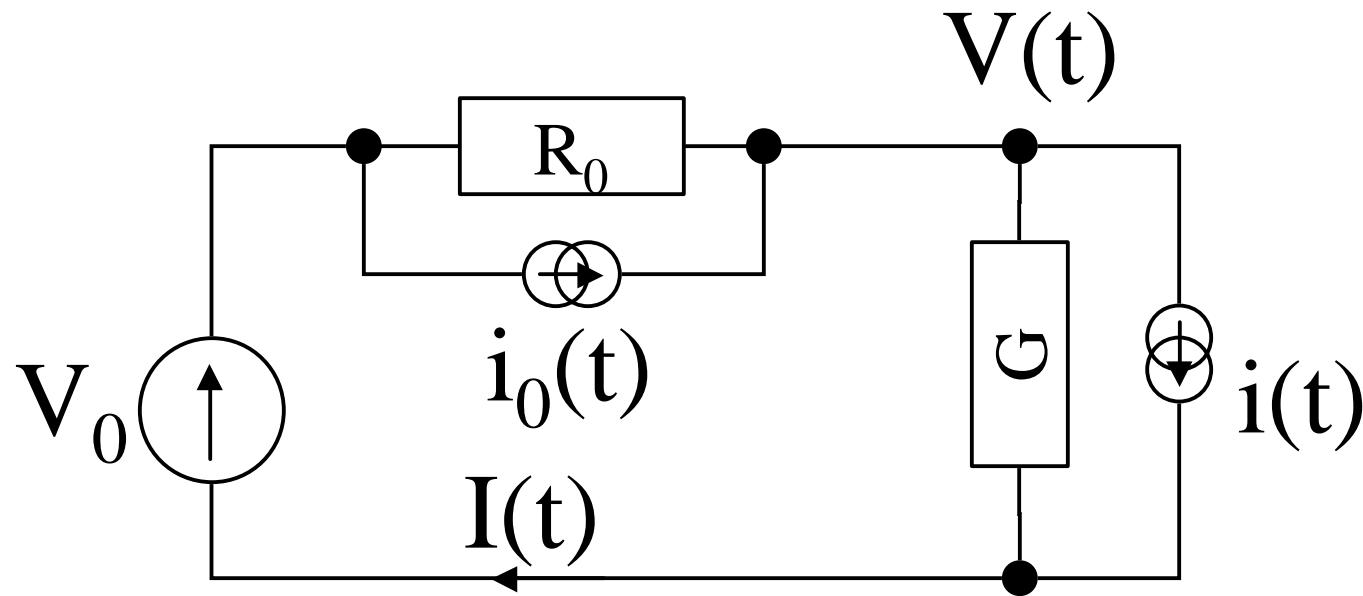
$$\begin{aligned} \langle \mathbf{d}I(t)^3 \rangle &= \iiint d\mathbf{w} d\mathbf{w}' d\mathbf{w}'' \overbrace{I(\mathbf{w})I(\mathbf{w}')I(\mathbf{w}'') \mathbf{d}(\mathbf{w} + \mathbf{w}' + \mathbf{w}'')}^{S_3(\mathbf{w}, \mathbf{w}', \mathbf{w}'')} \\ &\propto (\mathbf{w}_2 - 2\mathbf{w}_1)^2 S_3(0, 0) \quad \text{if } \mathbf{w}_2 > 2\mathbf{w}_1, \quad 0 \text{ otherwise} \end{aligned}$$

One must have a broadband detection: $\omega_2 > 2\omega_1$

Effect of bandwidth: result



Imperfect voltage bias



$$dV(t) = (R // R_0)(i_0 - i)$$

$$\langle dV \rangle^2 = (R // R_0)^2 (\langle i_0^2 \rangle + \langle i^2 \rangle - 2\langle i_0 i \rangle)$$

$$\langle dV \rangle^3 = (R // R_0)^3 (\langle i_0^3 \rangle - \langle i^3 \rangle + 3\langle i_0 i^2 \rangle - 3\langle i_0^2 i \rangle)$$

The probability distribution $P(i)$ depends on $V(t)$

Feedback and noise of the environment

Kindermann
Nazarov
Beenakker

$$\langle i_0 i^2 \rangle = \langle i_0 S_2(V(t)) \rangle \cong \langle i_0 S_2(V_{dc}) \rangle + \left\langle i_0 \frac{dS_2}{dV} dV(t) \right\rangle = \langle i_0^2 \rangle (R // R_0) \frac{dS_2}{dV}$$

Noise of the environment: T_{env}

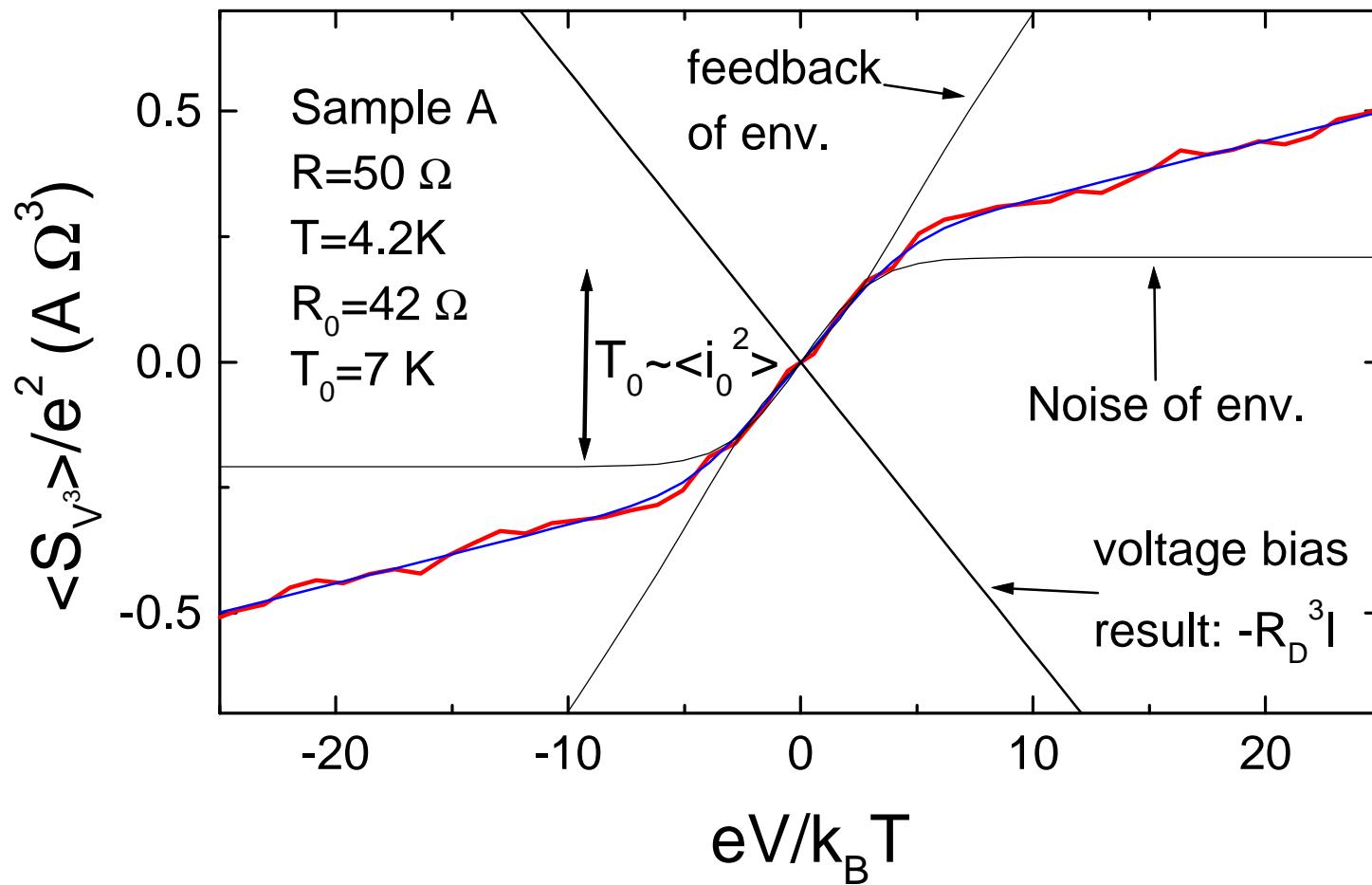
$$\langle i^3 \rangle = \langle i^3 \rangle_V + 3 \langle i S_2(V(t)) \rangle \cong \langle i^3 \rangle_V - 3 \langle i^2 \rangle (R // R_0) \frac{dS_2}{dV}$$

Feedback (even for $T_{\text{env}}=0$)

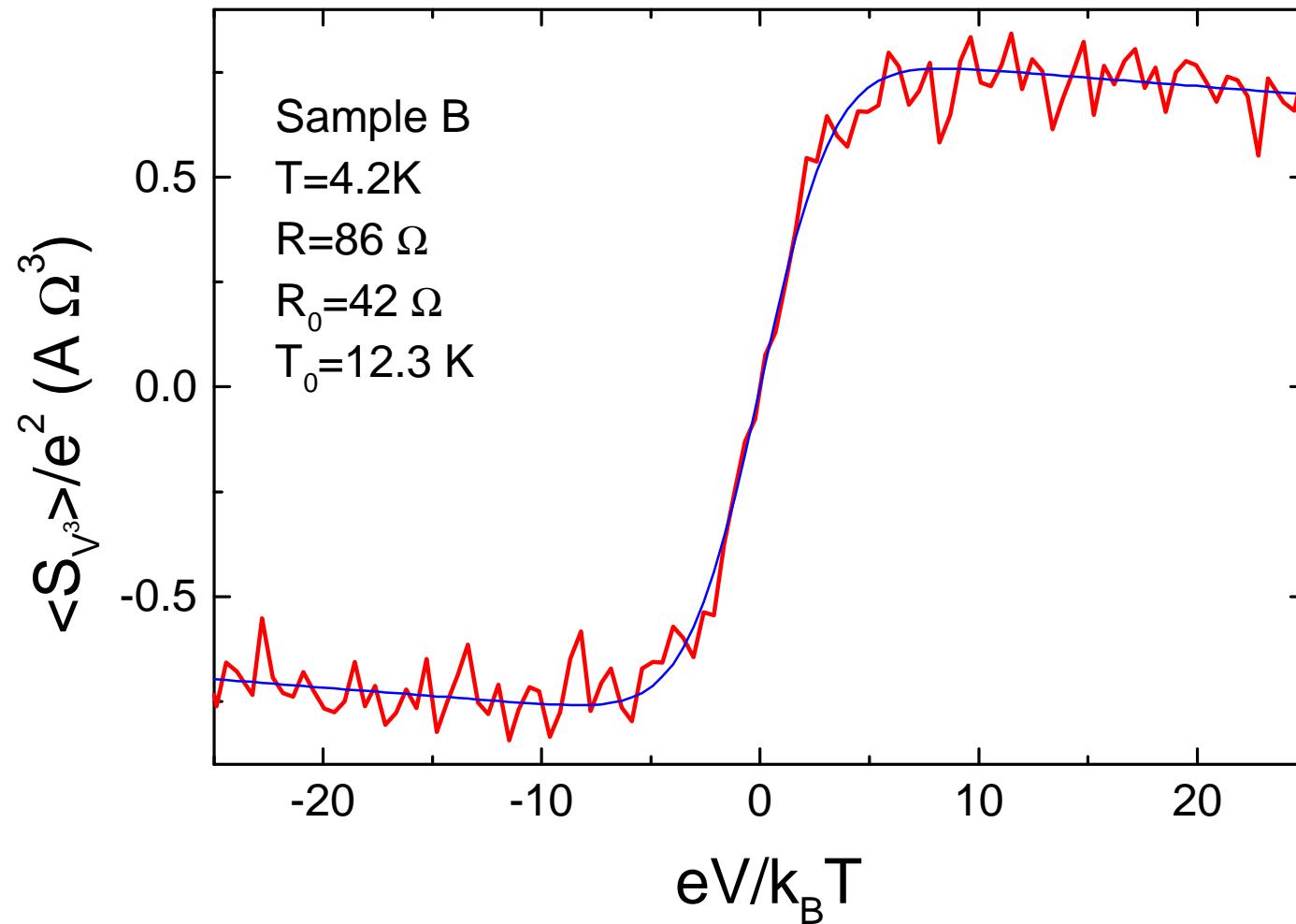
Bandwidth: junction: eV/h $\sim 10^{12}$ Hz
Environment: $(RC)^{-1} \sim 10^{10}$ Hz

Origin: e-e interactions (leads to Coulomb blockade)

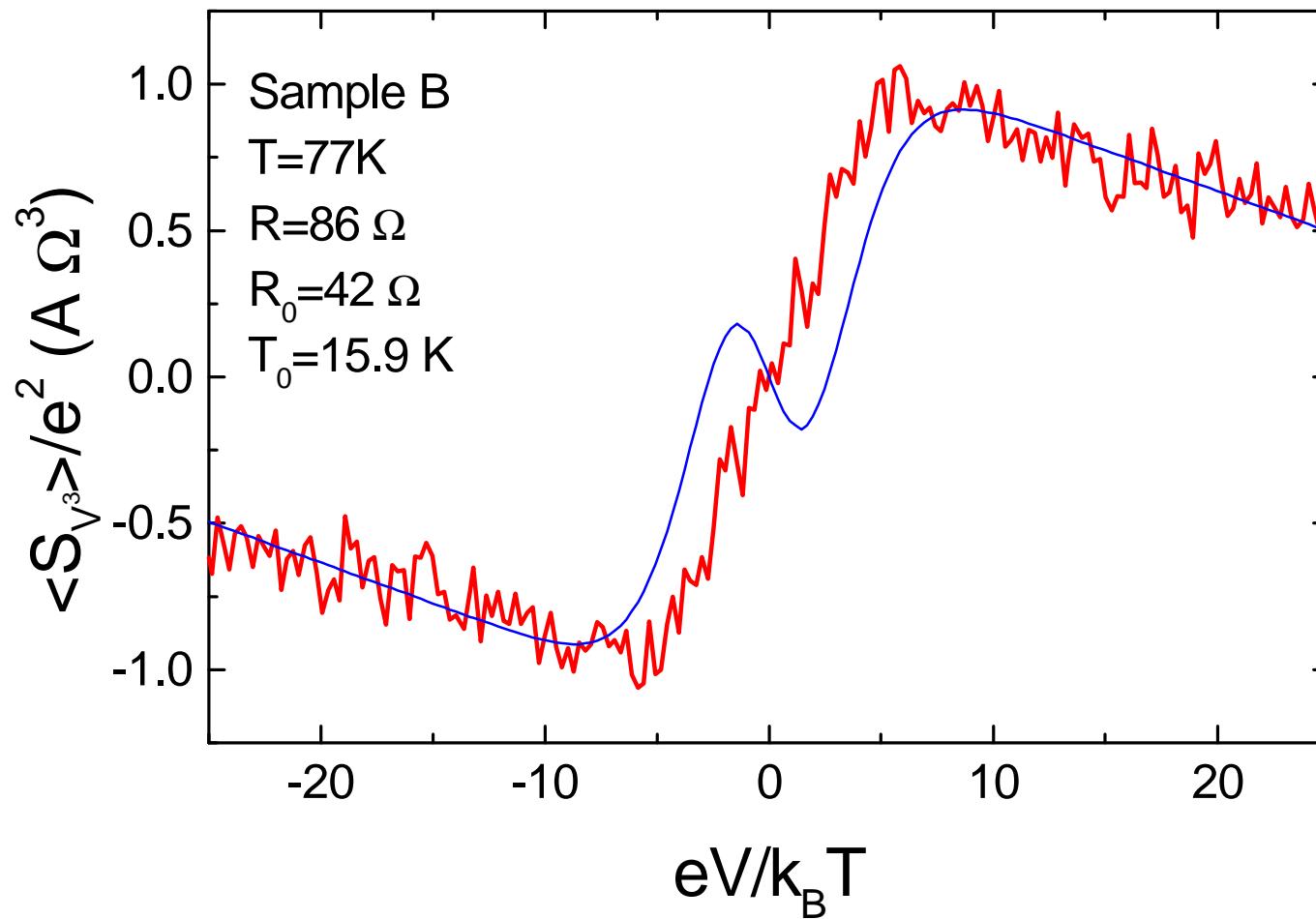
Exp. Result + theory: sample A



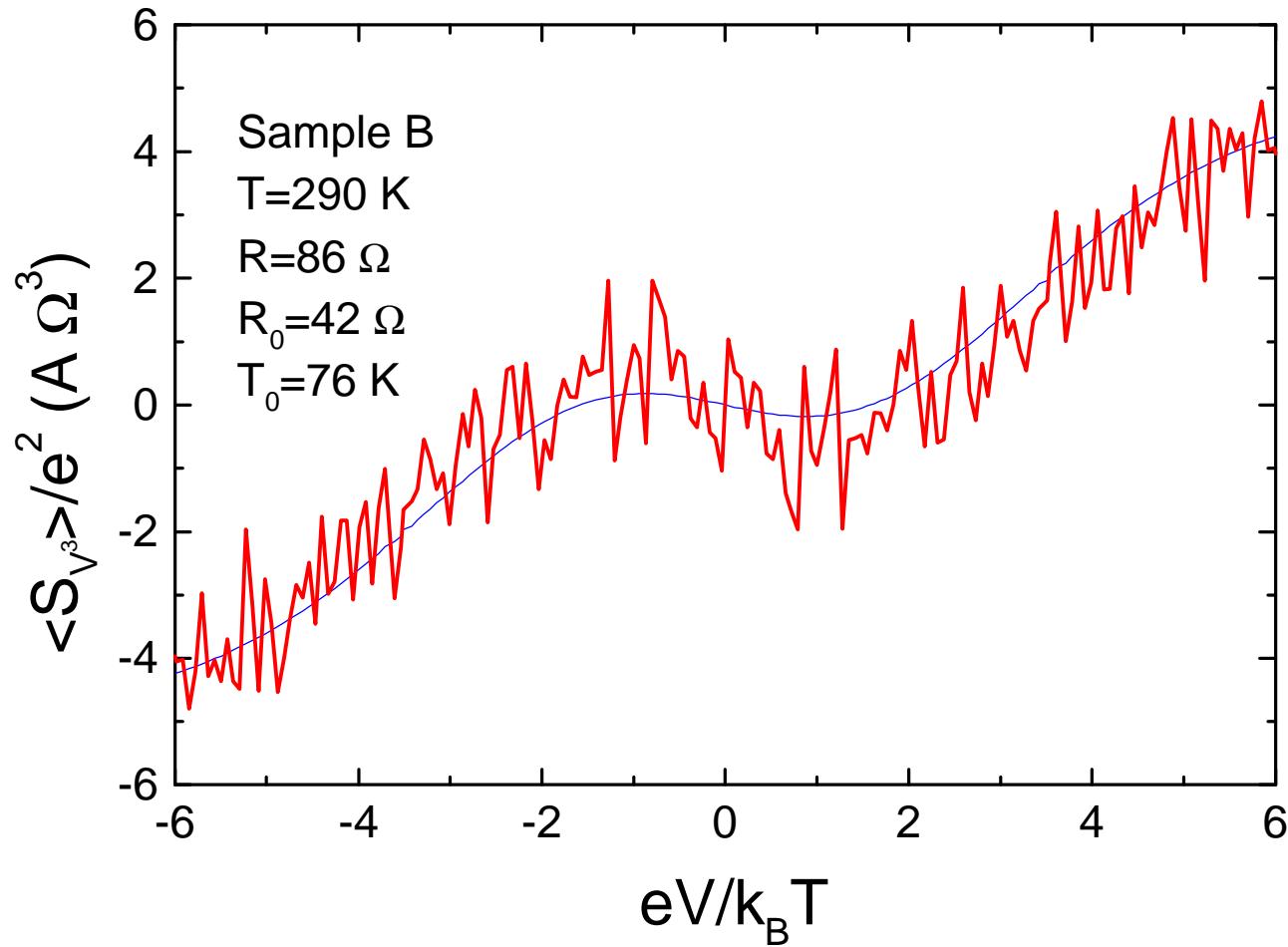
Exp. Result + theory: sample B



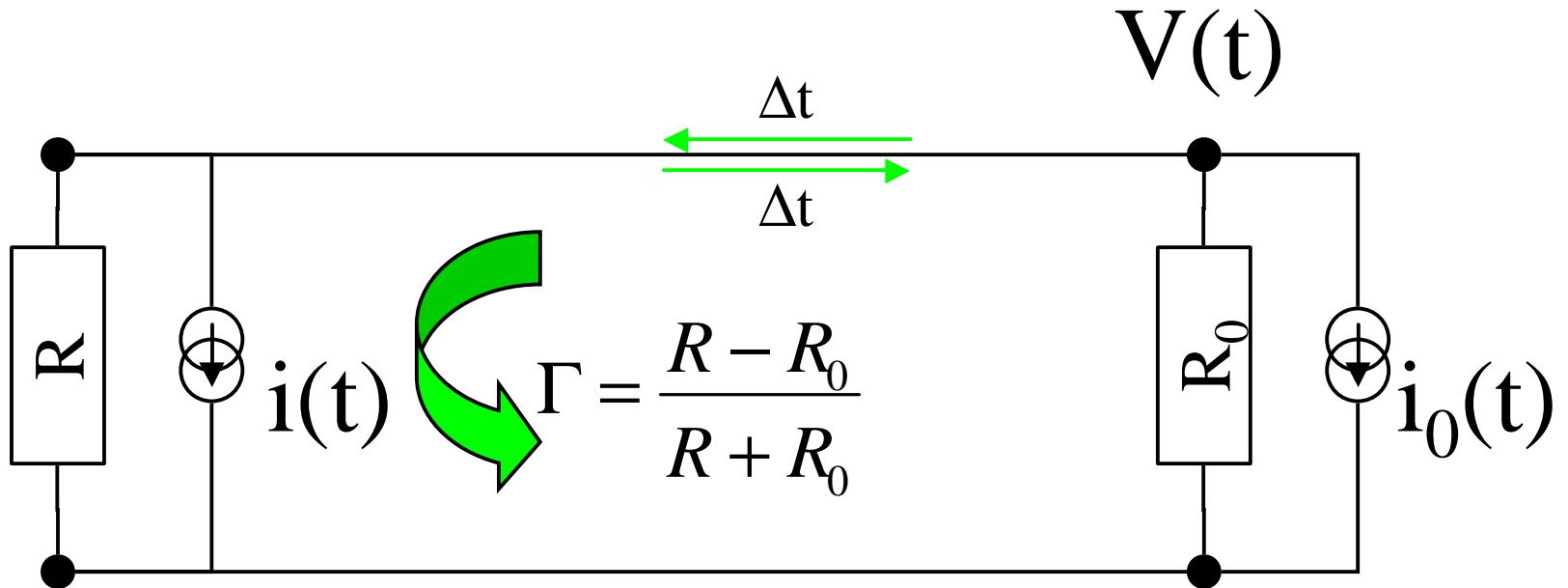
Exp. Result + theory: sample B



Exp. Result + theory: sample B



Effect of the propagation time



$$V(t) = \frac{1}{2} R_0 \left(i_0(t) + \Gamma i_0(t - 2\Delta t) \right) + \frac{1}{2} R (1 - \Gamma) i(t - \Delta t)$$

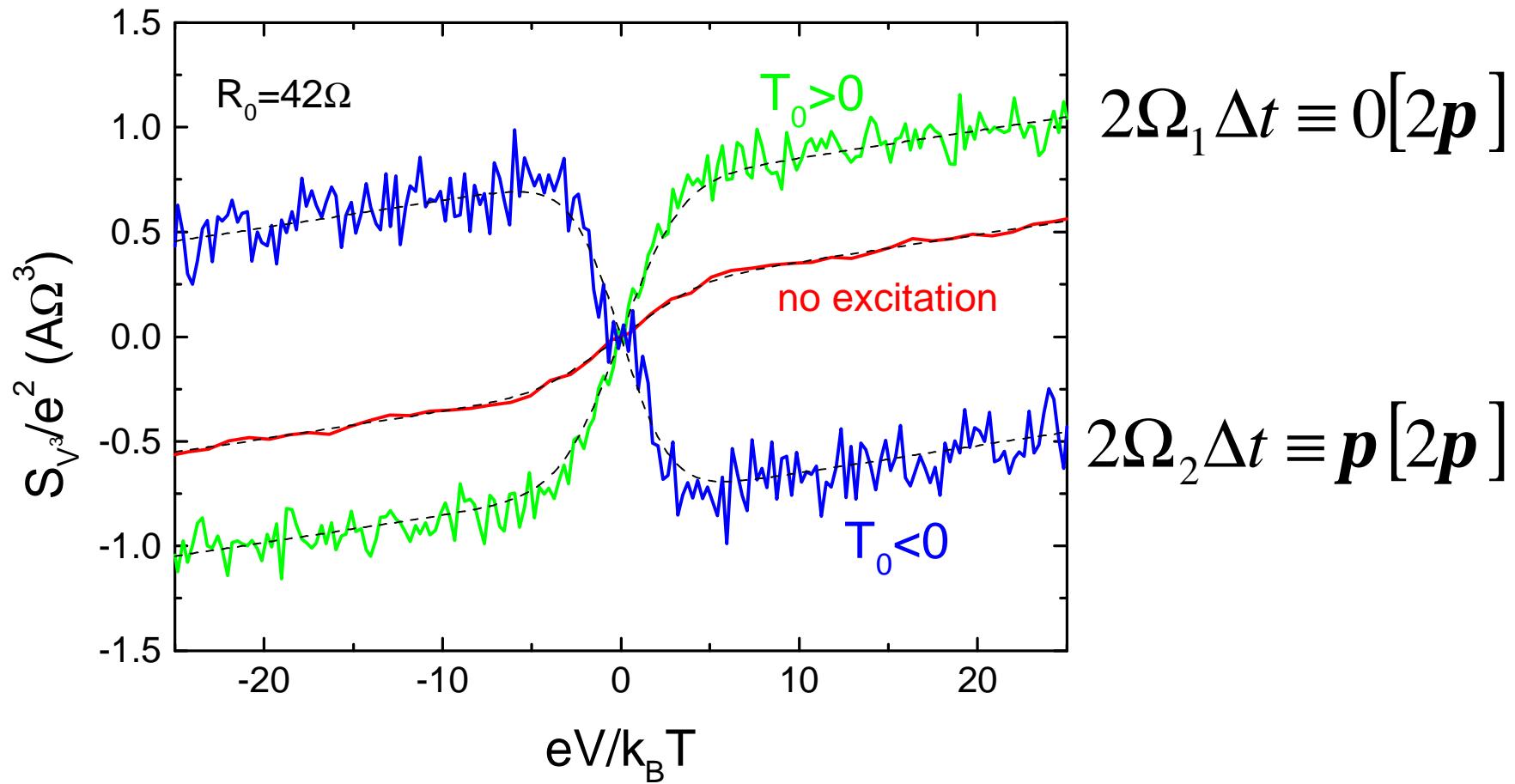
$$\langle i^2(t - \Delta t) \rangle \propto i_0(t - 2\Delta t)$$

$$\langle i_0(t) i_0(t - 2\Delta t) \rangle \approx 0$$

$$T_0^{\text{eff}} \sim \Gamma T_0$$

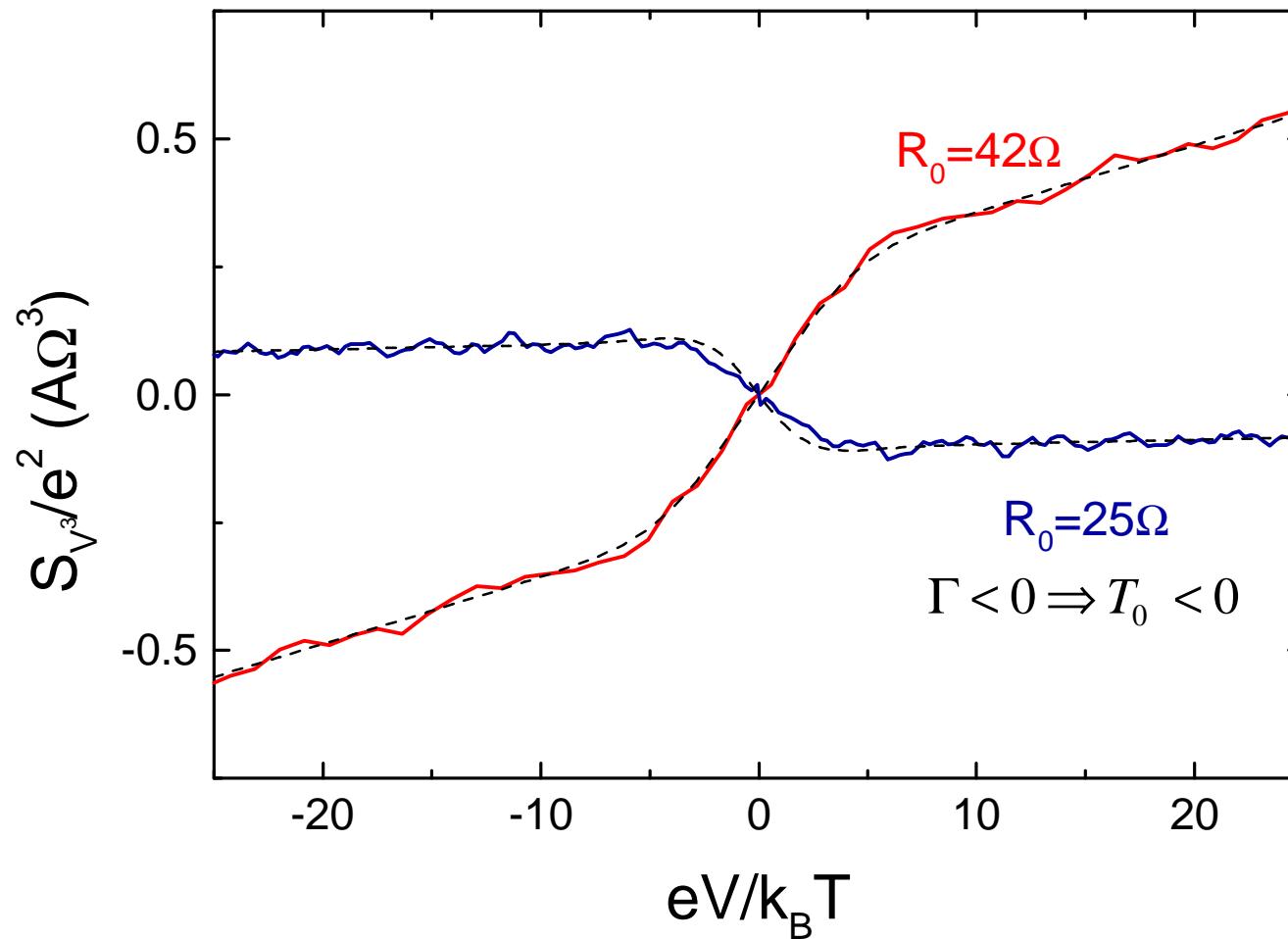
Noise of the environment

$$i_0(t) = A \cos \Omega t \quad T_0 \propto \langle i_0(t) i_0(t - 2\Delta t) \rangle \propto A^2 \cos 2\Omega \Delta t$$



Feedback of the environment

Change R_0 : add a resistor in parallel with the sample



Perspectives

- Good control of the environment
- Improve sensitivity to have access to more subtle effects
- Investigate any kind of good conductors: diffusive wire, NS interface, SNS, ...
- Phase transitions ?
- Carbon nanotubes ? Etc...