Interaction of solitons through radiation in optical fibers with randomly varying birefringence

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Propagation of solitons in optical fibers is studied taking into account the polarization mode dispersion (PMD) effect. We show that the soliton interaction caused by the radiation emitted by solitons due to the PMD disorder leads to soliton jitter, and we find its statistical properties. The theoretical predictions are justified by direct numerical simulations.

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Optical lines are widely used for transmission of information. In modern high-speed fiber communication the noise induced by optical amplifiers and the birefringent disorder are two major sources of transmission failure. Although the amplifier noise is short correlated in time, the birefringence is practically frozen. The birefringent disorder leads to fast random rotation of the principle axes of the polarization tensor along the fiber. Under certain conditions, this results in effective averaging of the nonlinearity in the signal propagation.[1–8] Then the signal is described by the Manakov equation.[9] A frequency dependence of birefringence leads to splitting the pulse into two polarization components, which results in pulse broadening known as polarization mode dispersion (PMD).[10–12] Since the first report of this phenomenon, the PMD effect has been studied extensively.[13–22]

In this Letter we investigate the influence of the PMD disorder on information transmission in the nonlinear regime when solitons are information carriers. We assume that the signal propagation is described by the Manakov equation and the PMD disorder is weak. In the presence of the PMD disorder, solitons are perturbed during propagation. This leads to soliton degradation[23] and soliton jitter.[24,25] Here we are interested in the effects related to the radiation emitted by solitons due to the PMD disorder. The radiation moves away from the soliton and influences other solitons, making it a mediator of the soliton interaction, which was first observed numerically in Ref. 26. Here we examine the statistical properties of this interaction.

In the reference system rotating together with randomly varying principal polarization axes one finds (after averaging over distances larger than the correlation length of the rotations) an equation for the envelope of the electromagnetic field[1–4]:

$$i\partial_z \mathbf{\Psi} + i\hat{m}(z)\partial_t \mathbf{\Psi} + \partial_t^2 \mathbf{\Psi} + 2|\mathbf{\Psi}|^2 \mathbf{\Psi} = 0 . \qquad (1)$$

Equation (1), written with proper dimensionless units, describes the signal propagation on scales larger than the birefringence correlation length. Here z is the position along the fiber and t is the retarded time. The envelope $\mathbf{\Psi}$ is a two-component complex field in which the components stand for different polarization states of the optical signal. The additive noise factor is omitted (this contribution leading to the Elgin–Gordon–Haus effect[27,28] can be examined separately). Equation (1) is the Manakov equation supplemented by an additional term responsible for the PMD effect. The PMD matrix $\hat{m}$ is a random Hermitian 2×2 traceless matrix. The birefringent disorder is stable at least on all the propagation-related time scales, i.e., $\hat{m}$ can be treated as t independent. It is expressed as $\hat{m} = h_1\hat{\sigma}_1 + h_2\hat{\sigma}_2 + h_3\hat{\sigma}_3$, where $\hat{\sigma}_i$ are Pauli matrices and h_i are real-valued functions of z. Since the correlation length of random fields h_i is short and observable quantities are expressed by integrals along the line of $h_i(z)$, one can apply the central limit theorem. Hence $h_i(z)$ can be treated as a Gaussian random process characterized by

$$\langle h_j \rangle = 0 , \qquad \langle h_j(z_1)h_k(z_2)\rangle = D\delta_{jk}\delta(z_1 - z_2) , \qquad (2)$$

where D represents the disorder intensity. We assume that PMD disorder is weak, i.e., $D \ll 1$. The isotropy of Eq. (2) is due to averaging over rotations of the principal axes.

The pure Manakov equation has exact solutions corresponding to solitons with different polarizations.[9] We assume that at the input to the line the perfect soliton profiles with a linear polarization are generated, corresponding to each bit 1. Then, during the signal propagation, the disorder perturbs the profile and leads to the emission of radiation by the solitons. The signal can be decomposed to a localized part (solitons) and a delocalized part (radiation). We assume that the solitons are well separated. Then, near a given soliton, the solution envelope $\mathbf{\Psi}$ can be written as

$$\mathbf{\Psi} = \exp(i\varphi)\left\{\frac{\eta \mathbf{e}}{\cosh[\eta(t - y)]} + \mathbf{v}\right\} . \qquad (3)$$

Here η and y are the amplitude and position of the soliton, respectively, and φ is a z- and t-dependent phase. The unit complex vector $\mathbf{e}$ characterizes the polarization of the soliton, and $\mathbf{v}$ describes radiation in its vicinity. Because of the weakness of the disorder, the radiation can be examined in the linear approximation. It is convenient to expand $\mathbf{v}$ over the eigenfunctions of the linearized Manakov equation near a given soliton pattern. To find the eigenfunctions, we use the Kaup perturbation technique[29] around each soliton and match the expressions of the eigenfunctions in the regions between the solitons. Then we find an explicit profile for $\mathbf{v}$ that is a superposition of contributions produced by sources proportional to h_j and localized at the solitons.

In the region $z \ll D^{-1}$, where the variations of the soliton amplitudes are negligible, the principal disorder effect is in fluctuations of the soliton position y related to the soliton interaction through radiation. Let us examine this effect assuming that the soliton polarization is $\mathbf{e} = (1, 0)$ and $\eta = 1$. An influence of the radiation on the soliton parameters can be found by projecting Eq. (1) onto the eigenmodes, corresponding to variations of the soliton parameters and radiation in Eq. (3). Then in the second-order approximation in $\mathbf{v}$ the equations describing variations of y are

$$\partial_z \beta = \mathcal{F}_{vv} + \mathcal{F}_{vh} + \Phi_{vv}\,, \tag{4}$$

$$\partial_z y = 2\beta + h_3 + \Pi + \mathcal{P}\,, \tag{5}$$

where β is the phase velocity of the soliton. The h_3 term in Eq. (5) corresponds to the direct jitter.[24,25] The influence of radiation is described by the second-order terms:

$$\mathcal{F}_{vv} = 2\int \mathrm{d}x \tanh x \cosh^{-2} x |v_2|^2,$$

$$\mathcal{F}_{vh} = -\mathrm{Im}\left[(h_1 - ih_2)\int \mathrm{d}x\, \frac{\tanh x}{\cosh x}\, \partial_t v_2\right],$$

$$\Pi = -\int \mathrm{d}x\, x \cosh^{-1} x\, \mathrm{Re}[(h_1 - ih_2)\partial_t v_2]\,,$$

$$\Phi_{vv} = \int \mathrm{d}x\, \frac{\tanh x}{\cosh^2 x}\,[4|v_1|^2 + v_1^2 + (v_1^*)^2]\,,$$

$$\mathcal{P} = i\int \mathrm{d}x \cosh^{-2} x [v_1^2 - (v_1^*)^2]\,, \tag{6}$$

where $x = t - y$ and v_1 and v_2 are polarization components of radiation field $\mathbf{v}$.

In two-soliton dynamics, radiation $\mathbf{v}$ is a superposition of two contributions from the solitons. Calculating these contributions, substituting the result into Eqs. (6), and then solving Eqs. (4) and (5), we find solutions corresponding to a given disorder h_j. Then the soliton position shifts δy_1 and δy_2 can be expressed as integrals of random processes. Hence, by the central limit theorem, the quantities δy_1 and δy_2 can be treated as Gaussian random variables asymptotically at large z. Statistical properties of such variables are completely characterized by their averages and variances. We find that the average values of δy_1 and δy_2 are negligible. Thus the main objects we need to calculate are mean-square fluctuations of the quantities, which determine the soliton jitter. Specifically, we examine two different cases: parallel and orthogonal polarizations. Averaging in accordance with Eq. (2), we obtain in the first case

$$\langle(\delta y_{1,2})^2\rangle = \frac{4}{3}\, G_{\|} D^2 z^3 + Dz\,, \tag{7}$$

$$\langle(\delta y_1 - \delta y_2)^2\rangle = \frac{8}{3}\,[1 + \cos(2\alpha)]G_{\|} D^2 z^3, \tag{8}$$

where $G_{\|} \approx 0.204$ and α is the phase mismatch of the solitons. The z^3 term of Eq. (7) corresponds to the radiation-mediated jitter, whereas the last term corresponds to the direct jitter; i.e., the radiation effect becomes dominant for $z \gg D^{-1/2}$. The correlation between the soliton displacements is explained by the strong correlation between the radiation emitted by solitons, which is related to the long correlation of the PMD disorder in time. For the orthogonal polarizations we find

$$\langle(\delta y_{1,2})^2\rangle = Dz\,, \qquad \langle(\delta y_1 - \delta y_2)^2\rangle = 4Dz\,. \tag{9}$$

In this case the radiation-mediated interaction gives zero contribution.

To confirm the theoretical predictions, extensive computational experiments are performed based on direct Monte Carlo simulation of Eq. (1). In Fig. 1 we plot the average square fluctuation $\langle(\delta y_1 - \delta y_2)^2\rangle$ of the intersoliton shift as a function of propagation length z for the parallel polarization. We chose the noise intensity $D = 0.0125^2$. For the sake of comparison with Eq. (8), three different phase mismatches $\alpha = 0, \pi/4, \pi/2$ are considered. For each α we average the fluctuations over 40 realizations. The solid curves stand for the numerical results, and the dashed curves stand for the theoretical predictions of Eq. (8). The same setup is used for the orthogonal polarization

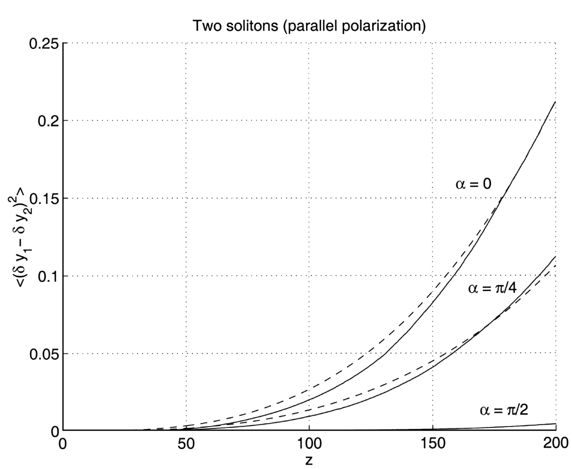

Fig. 1. Parallel polarization state: mean-square intersoliton shift as a function of z for three phase mismatches. Dashed curve, theory; solid curve, numerics.

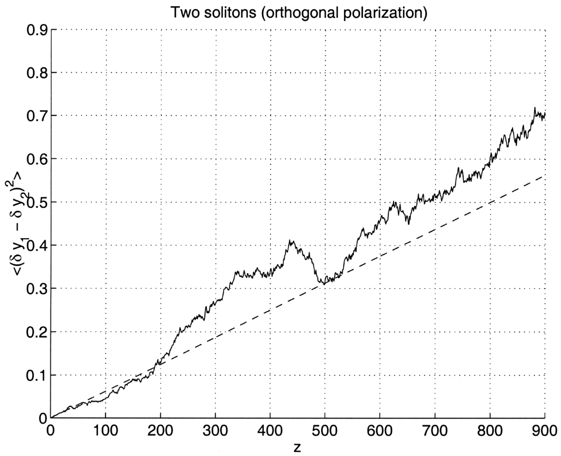

Fig. 2. Orthogonal polarization state: mean-square intersoliton shift as a function of z. Dashed line, theory; solid curve, numerics.

case. In Fig. 2 we plot $\langle(\delta y_1 - \delta y_2)^2\rangle$ and compare it with Eq. (9). The figures show reasonably good agreement between theory and numerics.

In the N-soliton case radiation $\mathbf{v}$ is a superposition of N contributions emitted by solitons. The principal terms on the right-hand side of Eqs. (4) and (5) (leading to major z contribution to soliton jitter) can also be written as a sum of N contributions originating from each soliton. This leads to the factor N in the expression for $\langle(\delta y_i)^2\rangle$. The average square of the intersoliton displacement is sensitive to their phase mismatch as in Eq. (8). However, the main contribution to the average is $\propto N$ at large N, and it is phase independent.

The effect examined in this Letter is the interaction between solitons mediated by their radiation. This leads to random displacements (jitter) of the solitons, which appear to be Gaussian random variables. In contrast with the nonintegrable case,[30] the systematic drift is zero, which is explained by the reflectiveless character of the radiation scattering on solitons in the integrable Manakov equation, similar to the case of random chromatic dispersion.[31] The jitter is independent of the soliton separation because of the 1d nature of the fiber. The dependence of the displacement variance on line length z and disorder strength D for the two-soliton case is determined by Eqs. (8) and (9) for the parallel and orthogonal polarizations, respectively. The jitter is suppressed for the orthogonal polarization and the phase mismatch $\alpha = \pi/2$. In the multisoliton case no such cancellation occurs and the typical displacement caused by the radiation-induced interaction is proportional to $z^{3/2}$ as for the Elgin–Gordon–Haus jitter.[27,28] Moreover, the displacement variance caused by the PMD increases $\propto N^{1/2}$ as the number of solitons N grows in the fiber, which can be potentially dangerous especially in long-distance high-speed communication systems.

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